Compact composition operators on real Lipschitz spaces of complex-valued bounded functions

Davood Alimohammadi *
Arak University

Sajedeh Sefidgar
Arak University

Abstract

We characterize compact composition operators on real Lipschitz spaces of complex-valued bounded functions on metric spaces, not necessarily compact, with Lipschitz involutions.

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1 Introduction and Preliminaries

Let X be a nonempty set, $V_{\mathbb{K}}(X)$ be a vector space over $\mathbb{K}$ of $\mathbb{K}$-valued functions on X and $T : V_{\mathbb{K}}(X) \longrightarrow V_{\mathbb{K}}(X)$ be a linear operator on X. If there exists a self-map $\phi : X \longrightarrow X$ such that $Tf = f \circ \phi$ for all $f \in V_{\mathbb{K}}(X)$, then T is call the composition operator on $V_{\mathbb{K}}(X)$ induded by ϕ.

Let X be a topological space. We denote by $C^b_{\mathbb{K}}(X)$ the set of all $\mathbb{K}$-valued bounded continuous functions on X. Then $C^b_{\mathbb{K}}(X)$ is a unital commutative Banach algebra over $\mathbb{K}$ under the pointwise operations and with the uniform norm

$$\| f \|_X = \sup\{|f(x)| : x \in X\} \quad (f \in C^b_{\mathbb{K}}(X)).$$

We denote by $C_{\mathbb{K}}(X)$ the algebra of all $\mathbb{K}$-valued continuous functions on X. Clearly, $C^b_{\mathbb{K}}(X) = C_{\mathbb{K}}(X)$ whenever X is compact. We write $C^b(X)$ and $C(X)$ instead of $C^b_{\mathbb{C}}(X)$and $C_{\mathbb{C}}(X)$, respectively.

Let (X, d) and (Y, ρ) be metric spaces. A map $\phi : X \longrightarrow Y$ is called a Lipschitz mapping from (X, d) into (Y, ρ) if there exists a constant $M \geq 0$ such that $\rho(\phi(x), \phi(y)) \leq Md(x, y)$ for all $x, y \in X$. A map $\phi : X \longrightarrow Y$ is called supercontractive from (X, d) into (Y, ρ) if

$$\lim_{d(x,y)\to 0} \frac{\rho(\phi(x), \phi(y))}{d(x, y)} = 0.$$

Let (X, d) be a metric space. A function $f : X \longrightarrow \mathbb{K}$ is called a $\mathbb{K}$-valued Lipschitz function on (X, d) if f is a Lipschitz mapping from (X, d) into the Euclidean metric space

*Speaker