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Weakly spatial frames

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Abstract

The aim of this paper is to determine weakly spatial frames. The concept of weakly spatiality is actually weaker than spatiality and they are equivalent in the case of regular frames. For compact conjunctive frames, the notion of spatiality, weak spatiality and dual atomicity coincide.

Keywords: Frame, Weakly spatial frame, Regular frame, spatial frame, Ring of real-valued continuous functions on a frame
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1 Introduction

Throughout, L denotes a frame, ΣL denotes the set of prime elemets of L, and $\mathcal{R}L$ denotes the ring of real-valued continuous functions on L.

In the theroy of frames (or "pointfree topology"), several authors have tried to find a suitable form of separation axioms. In [6], T_2- frames are describe also authors investigate almost compact frames and H-closed extensions of T_2- frames. All unexplained facts concerning separation axioms can be found in [6] or in [7].

The concept of a weakly spatial frame is introduced and the main results of the note are given in Section 2. The weakly spatial frames play an important role in this note. For conjunvtive frames, they are equivalent with spatial frames. There are many examples of frames which are weakly spatial but they are not spatial (Remark 2.3). Using the Axiom of Choice, compact frames are weakly spatial (Proposition 2.4).

Let L be a weakly spatial frame. In Proposition 2.8, it is proved that if $\alpha \in \mathcal{R}L$ and $\Sigma_{coz(\alpha)} = \emptyset$, then $coz(\alpha) = \perp$, i.e., $\alpha = \mathbf{0}$. Also for every $\alpha \in \mathcal{R}L$, $Z(\alpha) = \emptyset$ if and only if $coz(\alpha) = \top$, i.e., α is a unit of $\mathcal{R}L$(Proposition 2.10). Finally, in the last proposition, it is shown that ΣL is a compact space if and only if L is a compact frame.

Here, we recall some definitions and results from the literature on frames and the pointfree version of the ring of continuous real valued functions. For more details see the appropriate references given in [1, 5, 7].

A *frame* is a complete lattice M in which the distributive law $x \wedge \bigvee S = \bigvee\{x \wedge s : s \in S\}$ holds for all $x \in L$ and $S \subseteq M$. We denote the top element and the bottom element of M

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