On V-regular semigroups

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Abstract

A regular semigroup S is V-regular if $V(ab) \subseteq V(b)V(a)$, for all $a, b \in S$. A characterization of a V-regular semigroup is given. Congruences on V-regular semigroups are described in terms of certain congruence pairs.

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1 Introduction

A regular semigroup S is called V-regular, if $V(ab) \subseteq V(b)V(a)$ for all $a, b \in S$. This concept was introduced by Onstad [4]. This class of semigroups is dual to orthodox semigroups, namely, regular semigroups satisfy that $V(b)V(a) \subseteq V(ab)$ for all elements $a, b \in S$. Properties of V-regular semigroups were given by Nambooripad and Pastijn in [3]. Congruences on regular semigroups have been explored extensively. The kernel-trace approach is an effective tool for handling congruences on regular semigroups, which had been investigated by many authors. The purpose of this paper is to give a characterization of a V-regular semigroup, and to describe congruences on V-regular semigroups in terms of certain congruence pairs. We refer the reader to [2] for basic definitions and terminology relating to semigroups and monoids. If S is a regular semigroup, $a \in S$, then $V(a)$ denotes the set of inverses of a in S. The set of idempotents of S is denoted by $E(S)$. On $E(S)$, we define the natural partial order ω given by $e\omega f \Leftrightarrow ef = fe = e$. For $e, f \in E(S)$, $S(e, f) = fV(ef)e$, is the sandwich set of e and f. The following simple statements will be applied without further mention: for $e, f \in E(S)$,

$$eLf \Rightarrow S(e, f) = f,$$

$$eRf \Rightarrow S(e, f) = e.$$

If ρ is a congruence on S and $h \in S(e, f)$, then $h\rho \in S(e\rho, f\rho)$. Let τ be a relation on S. The congruence generated by τ is denoted by τ^*. If γ is an equivalence on S, then γ^0 is the greatest congruence on S contained in γ. $C(S)$ is the lattice of congruences on S.

Lemma 1.1 (3). *A regular semigroup S is V-regular if and only if the partial band $(E(S), \circ)$ determined by S satisfies the following:*

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