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A note on the transitive groupoid spaces

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Abstract

If a group G acts on a set X and H is a subgroup of G, the Frattini argument shows that H acts transitively on X if and only if G acts transitively on X and $G = HStab_x$ for some $x \in X$, where $Stab_x$ is the stabilizer of x in G. There is another useful result in group action which indicates that the action of G on a set X is doubly transitive if and only if, for each $x \in X$, the group $Stab_x$ acts transitively on $X \setminus \{x\}$, where the cardinal number of X is more than two. In this paper if a groupoid acts on a set X, then by using sections, special subsets of X, instead of the points of X in the group case, we will extend these results to the groupoid case.

Keywords: Groupoid; Groupoid space; Frattini argumen
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1 Introduction

When a group G acts on a set X, the point stabilizer of $x \in X$ is denoted by $Stab_x$ and is a subgroup of G. In the case where G acts transitively on X, then the stabilizers $Stab_x(x \in X)$ form a single conjugacy class of subgroups of G. The Frattini argument indicate that a subgroup H of G acts transitively on X if and only if $G = HStab_x$ for some $x \in X$ [1]. The action of the group G on the set X is naturally extend to an action of G on the cartesian product $X \times X$ by $g.(x, y) = (g.x, g.y)$. The action of G on X is called doubly transitive, if for two pairs $(x_1, x_2), (y_1, y_2)$ in $X \times X$ with $x_1 \neq x_2, y_1 \neq y_2$, there exists $g \in G$ with $g.x_1 = y_1, g.x_2 = y_2$. The action of G on X is doubly transitive if and only if, for each $x \in X$, the group $Stab_x$ acts transitively on $X \setminus \{x\}$ [1].

A groupoid (see definition 1.1 of [4]) is a set G endowed with a product map $(x, y) \mapsto xy : G^2 \to G$ where G^2 as a subset of $G \times G$ is called the set of *composable pairs*, and an inverse map $x \mapsto x^{-1} : G \to G$ such that the following relations are satisfied:

1. For every $x \in G$, $(x^{-1})^{-1} = x$.
2. If $(x, y), (y, z) \in G^2$, then $(xy, z), (x, yz) \in G^2$ and $(xy)z = x(yz)$.
3. For all $x \in G$, $(x^{-1}, x) \in G^2$ and if $(x, y) \in G^2$, then $x^{-1}(xy) = y$. Also for all $x \in G$, $(x, x^{-1}) \in G^2$ and if $(z, x) \in G^2$, then $(zx)x^{-1} = z$.

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