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Some properties of fundamentally nonexpansive mappings in CAT(0) spaces

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Abstract

The purpose of this paper is to give a fixed point theorem and a convergence theorem for fundamentally affine nonexpansive self-mappings in a complete CAT(0) space. Specially, we show that the fixed points set of such mappings defined on a nonempty bounded closed convex subset of a a complete CAT(0) space is always nonempty and closed.

Keywords: Affine mapping, Fixed point, Fundamentally nonexpansive mapping, CAT(0) space
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1 Introduction and preliminares

Let K be a nonempty subset of a metric space (X, d), and let $T : K \to K$ be a mapping. We denote by $F(T)$ the set of fixed points of T, i.e., $F(T) = \{x \in K : Tx = x\}$. The mapping T is said to be
(i) nonexpansive if

$$d(Tx, Ty) \leq d(x, y) \text{ for all } x, y \in K;$$

(ii) fundamentally nonexpansive if

$$d(T^2x, Ty) \leq d(Tx, y) \text{ for all } x, y \in K;$$

(iii) quasi-nonexpansive if $F(T)$ is nonempty and

$$d(Tx, u) \leq d(x, u) \text{ for all } x \in K \text{ and } u \in F(T).$$

It is evident that fundamental nonexpansiveness is weaker than nonexpansiveness and stronger than quasi-nonexpansiveness.

Fixed point theory in a CAT(0) space was first studied by Kirk (see [6] and [7]). He showed that every nonexpansive mapping defined on a nonempty bounded closed convex subset of a a complete CAT(0) space always has a fixed point. Since then the fixed point theory for single-valued and multivalued mappings in CAT(0) spaces has been rapidly developed and many papers have been appeared. The aim of this paper is to present

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