A preconditioner based on the shift-splitting method for generalized saddle point problems

Davod Khojasteh Salkuyeh
University of Guilan

Mohsen Masoudi*
University of Guilan

Davod Hezari
University of Guilan

Abstract

In this paper, we propose a preconditioner based on the shift-splitting method for generalized saddle point problems with nonsymmetric positive definite (1,1)-block and symmetric positive semidefinite $(2,2)$-block. The proposed preconditioner is obtained from an basic iterative method which is unconditionally convergent. We also present a relaxed version of the proposed method. Some numerical experiments are presented to show the effectiveness of the method.

Keywords: Generalized saddle point, preconditioner, shift-splitting, Navier-Stokes.
Mathematics Subject Classification [2010]: 65F10, 65F50

1 Introduction

We consider the solution of the following large and sparse generalized saddle point problem

$$\mathcal{A}u = \begin{pmatrix} A & B^T \\ -B & C \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} f \\ -g \end{pmatrix} = b, \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$ is nonsymmetric positive definite ($x^T Ax > 0$ for all $0 \neq x \in \mathbb{R}^n$), $C \in \mathbb{R}^{m\times m}$ is symmetric positive semidefinite, the matrix $B \in \mathbb{R}^{m\times n}$ is of full row rank, $x, f \in \mathbb{R}^n$, $y, g \in \mathbb{R}^m$ and $m \leq n$. It can be verified that the system (1) has a unique solution [1, Lemma 1.1]. Saddle point problems of the form (1) arise from finite difference or finite element discretization of the Navier-Stokes problem (see [2] and references therein). Several iterative method have been presented to solve system (1) or some special cases of it in the literature. The main methods have been reviewed in [2]. In [1], Benzi and Golub presented the Hermitian and skew-Hermitian splitting (HSS) method to solve (1). Since, in general, the HSS method is too slow to be used to solve (1), they used the GMRES method in conjunction with the preconditioner extracted from the HSS method to solve (1). Recently, when the matrix A is symmetric positive definite, Salkuyeh et al. in [6] have presented a stationary iterative method based on the shift-splitting method to solve (1). The proposed method naturally serves a preconditioner for the problem (1). More recently, Cao et al. in [3] have considered the same iterative method to solve the system (1) when $C = 0$. In this paper, we consider the problem (1) in its general form and investigate the convergence properties of the proposed iterative method and the corresponding preconditioner.

*Speaker