Spectrum and Eigenvalues of Quaternion Matrices

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Abstract

In this paper we introduce left and right eigenvalues for quaternion-valued matrix Q. Also, we will show that the spectrum of Q is not the set of its eigenvalues.

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1 Introduction

The study of inequalities for compact operators, especially operators acting upon finite-dimensional spaces, is frequently carried out through an analysis of the eigenvalues or singular values. For matrices with entries in a general ring $\mathcal{R}$ there is no theory of eigenvalues. However, if the ring $\mathcal{R}$ is an algebra over algebraically closed field, then existance of eigenvalues can be proved.

The real quaternion algebra $\mathbb{H}$ is known as a four dimensional vector space over the real number field $\mathbb{R}$ with its basis $\{1, i, j, k\}$ satisfying the multiplication laws

$$\begin{array}{lll} i^2 = j^2 = k^2 = -1 & , \quad ijk = -1 & \\ ij = -ji = k & , \quad jk = -kj = i & , \quad ki = -ik = j \end{array}$$

and 1 acting as unity element. In this case any element in $\mathbb{H}$ can be written as $q = a_0 + a_1 i + a_2 j + a_3 k$ where $a_j's$ are all real numbers.
We shall always write every quaternion q in the form $q = z_1 + z_2 j$ where $z_1 = a_0 + a_1 i$ and $z_2 = a_2 + a_3 i$ are complex numbers.
A quaternion matrix Q therefore can be written $Q = A_1 + A_2 j$, where $A_1$and A_2 are unique complex matrices. The function $\phi : M_n(\mathbb{H}) \to M_{2n}(C)$ then defined by

$$\phi(Q) = \begin{bmatrix} A_1 & -\overline{A_2} \\ A_2 & \overline{A_1} \end{bmatrix}$$

is an injective $*$-homomorphism. The matrix $\phi(Q)$ is called the complex representation of Q.
Various operation properties on complex representation of quaternion matrices can easily be proved:

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