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A spectral method for the solution of KdV equation via orthogonal rational basis functions

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Abstract

In this paper, a set of orthogonal rational Chebyshev functions in $L^2(0, +\infty)$ is generated by the orthogonal Chebyshev polynomials. Moreover a new computational method based on these new basis functions is proposed for solving KdV equations on the semi–infinite interval with initial–boundary conditions. In this way, a weak formulation for the above mentioned problems is obtained, and also a Galerkin method using these basis functions is applied. Some numerical examples are included for demonstrating the efficiency of the method.

Keywords: Partial differential equations, KdV equation, Spectral methods, Chebyshev polynomials, Orthogonal rational Chebyshev functions.
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1 Introduction

In 1895 Kurteweg and de Vries proposed the equation

$$u_t + uu_x + u_{xxx} = 0 \tag{1}$$

as a model for water waves in shallow regions. This equation, which has been known as KdV equation [8], is a well-known equation in the field of nonlinear waves. In 1965 Zabusky and Kruskal used the leap-frog method for discretizing the KdV equation [9].Two years later, Gardner, Greene, Kruskal and Miura discovered that assuming the solutions decay at infinity with sufficient rates, equation (1) can be efficiently solved via a method called the Inverse Scattering Method [5]. In 1982, Christov [4] and Boyd [3, 2] developed some spectral methods on infinite intervals by using orthogonal systems of rational functions. In 2000, Guo [7] developed a rational spectral method based on a weighted orthogonal system consisting of rational function built from Legendre polynomials with a rational transformation. Recently, Zhang and Ma [10] proposed a combined Petrov-Galerkin scheme using orthogonal Legendre rational functions for solution of the following problem:

$$\begin{cases} u_t + uu_x + u_{xxx} = f(x,t)\ ,\ x \in [0, +\infty]\ ,\ t \in (0, T] \\ u(0,t) = \lim\limits_{x \to +\infty} u(x,t) = \lim\limits_{x \to +\infty} u_x(x,t) = 0\ , \\ u(x,0) = u_0(x). \end{cases} \tag{2}$$

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