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A Modified Reduced Differential Transform Method to Delay Differential Equations Using Padé Approximation

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Abstract

In this paper we present an application of technique combining Reduced Differential Transform Method (RDTM), Laplace transform and Padé approximant to find the analytical solutions for DDEs. Solutions to DDEs are first obtained in convergent series form using the RDTM. To improve the solution obtained from RDTM's truncated series, we apply Laplace transform to it, then convert the transformed series into a meromorphic function by forming its Padé approximant. Finally, we take the inverse Laplace transform of the Padé approximant to obtain the analytical solution.

Keywords: Reduced differential transform method, Delay differential equation, Laplace transform, Padé approximant, Laplace-Padé resummation method
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1 Introduction

In this section we will explain the basic definitions of DDE, RDTM, Padé approximant, Laplace-Padé resummation.

Definition 1.1: We define the nth order delay differential equations (DDE) of the form as follows:

$$u^{(n)}(x) = f(x, u(x), u(\eta_1(x)), u(\eta_2(x)), \cdots, u(\eta_m(x))), \quad x \in I = [0, a], \tag{1}$$

where $u : I \to R$, $f : I \times R^2 \to R$, $\eta_i : I (i = 1, 2, \cdots, m)$ and $\eta_i(x) < x$ for $x \in I$.
The basic definitions of RDTM [?] are defined as follows.

Definition 1.2: If function $u(x, t)$ is analytic and differentiated continuously with respect to time t and space x in the domain of interest, then let

$$U_k(x) = \frac{1}{k!}[\frac{\partial^k}{\partial t^k} u(x, t)]_{t=0}, \tag{2}$$

where the t-dimensional spectrum function $U_k(x)$ is the transformed function. In this paper, the lowercase $u(x, t)$ represents the original function while the uppercase $U_k(x)$ stands for the transformed function.

Definition 1.3: The differential inverse transform of $U_k(x)$ is defined as follows:

$$u(x, t) = \sum U_k(x) t^k, \tag{3}$$

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