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Chebyshevity and proximity in quotient spaces

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Abstract

We obtain a sufficient and necessary theorems simple for Chebyshevity of the best approximate sets in quotient spaces. Approximation theory, which mainly consists of theory of nearest points (best approximation) and theory of farthest points (worst approximation), is an old and rich branch of analysis. The theory is as old as Mathematics itself. The ancient Greeks approximated the area of a closed curve by the area of a polygon.

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1 Introduction

In this paper with a new ways we obtain some results on quotient spaces about proximinality, Chebyshevity of approximate sets.

Let W be a non-empty subset of a normed linear space X. For any $x \in X$, the (possibly empty) set of best approximations x from M is defined by

$$P_W(x) = \{y \in W : \|x - y\| = d(x, W)\},$$

where $d(x, W) = \inf\{\|x - y\| : y \in W\}$, and

$$\widehat{W} = \{x \in X : \|x\| = d(x, W)\}.$$

The subset W is said to be proximinal if the set $P_W(x)$ is non-empty for every $x \in X$ and the set W is Chebyshev if $P_W(x)$ is a singleton set. The closed unit ball of X is B_X and

$$B_X = \{x \in X : \|x\| \leq 1\}$$

Let W be a subspace of a normed space X. We define the quotient space X/W to be the set of all cosets $x + W$ of W together with the following operations:

$$(x + W) + (y + W) = (x + y) + W,$$

and

$$\lambda(x + W) = \lambda x + W,$$

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