GENERALIZATIONS OF PRIMARY SUBMODULES

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Abstract

Let R be a commutative ring with $1 \neq 0$ and M be a unitary R-module. Let $S(M)$ be the set of all submodules of M. In this paper, we extend the concept of 2-absorbing primary submodules to the context of ϕ-2-absorbing primary submodules. Let $\phi : S(M) \longrightarrow S(M) \cup \emptyset$ be a function. A proper submodule N of M is said to be a ϕ-2-absorbing primary submodule of M if whenever $a, b \in R$ and $x \in M$ with $abx \in N \setminus \phi(N)$ implies $ab \in (N : M)$ or $ax \in rad(N)$ or $bx \in rad(N)$. Anumber of results concerning ϕ-2-absorbing primary submodules are given.

Keywords: primary submodule, 2-absorbing submodule, 2-absorbing primary submodule, ϕ-primary submodule, ϕ-2-absorbing primeary submodule.
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1 Introduction

Throughout this paper R denotes a commutative ring with $1 \neq 0$ and M denotes a unitry R-module and the set of all submodules of M is denoted by $S(M)$. A submodule N of M is said to be proper if $N \neq M$. Let N be a submodule of M. Then $(N : M) = \{r \in R | rM \subseteq N\}$ is an ideal of R.

One of the natural generalisations of prime ideals which have attracted the interest of several authors in the last two decades is the notion of prime submodules, (see for example [1],[3-6]).Generalizations of prime submodules to the context of ϕ-prime submodules are studied extensively in [2], [7], [8]. Recall that a proper submodule N of M is called a 2-absorbing submodule of M as in [2] if whenever $abx \in N$ for some $a, b \in R$ and $x \in M$, then $ab \in (N : M)$ or $ax \in N$ or $bx \in N$. A proper submodule N of M is called a weakly prime submodule of M as in [7] if whenever $0 \neq ax \in N$ for some $a \in R$ and $x \in M$, then $a \in (N : M)$ or $x \in N$. We say that a proper submodule N of M is a weakly primary submodule of M if whenever $0 \neq ax \in N$ for some $a \in R$ and $x \in M$, then $a \in (N : M)$ or $x \in rad(N)$.

Also, we say that a proper submodule N of M is a 2-absorbing primary submodule of M if whenever $a, b \in R$ and $x \in M$ with $abx \in N$, then $ab \in (N : M)$ or $ax \in rad(N)$ or $bx \in rad(N)$. A proper submodule N of M is a weakly 2-absorbing primary submodule of M if whenever $a, b \in R$ and $x \in M$ with $0 \neq abx \in N$ implies $ab \in (N : M)$ or $ax \in rad(N)$ or $bx \in rad(N)$. Recall that a proper submodule N of M is called a ϕ-2-absorbing submodule of M as in [2] if whenever $a, b \in R$ and $x \in M$ with $abx \in N \setminus \phi(N)$

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