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2-absorbing Submodules and Flat modules *

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Abstract

2-absorbing submodule is generalization of the notion of 2-absorbing ideal. We will study 2-absorbing submodules and we prove that 2-absorbing submodules are not too far from prime submodules, which are well-known and studied concepts. Also we find some properties of 2-absorbing submodules in flat modules.

Keywords: 2-absorbing submodule, Flat modules, Faithfully flat modules
Mathematics Subject Classification [2010]: 13E05, 13C99, 13C13, 13F05, 13F15.

1 Introduction

Throughout this paper all rings are commutative with identity and all modules are unitary. Also we consider $n > 1$ a positive integer. Let N be a submodule of an $R-$module M. The set $\{r \in R | rM \subseteq N\}$ is denoted by $(N : M)$. Also we consider $T(M) = \{m \in M | \exists 0 \neq r \in R,\ rm = 0\}$. A module M is called torsion-free, if $T(M) = 0$.

According to [1] an ideal I of a ring R is called *2-absorbing*, if $abc \in I$ for $a, b, c \in I$ implies that $ab \in I$ or $bc \in I$ or $ac \in I$.

A module version of 2-absorbing ideals is introduced as follows:

Definition 1.1. A proper submodule N of M will be called 2-absorbing if for $r, s \in R$ and $x \in M$, $rsx \in N$ implies that $rs \in (N : M)$ or $rx \in N$ or $sx \in N$.

In order to obtain our main results, we use some definitions and lemma such as the following:

Let F be an R-module. Writing φ to stand for a sequence $... \longrightarrow N' \longrightarrow N \longrightarrow N'' \longrightarrow ...$ of R-modules and linear maps, we let $F \otimes \varphi$ stand for induced sequence $... \longrightarrow F \otimes N' \longrightarrow F \otimes N \longrightarrow F \otimes N'' \longrightarrow ...$

The R-module F is called flat, if for every sequence φ,

$$\varphi \text{ is exact} \Longrightarrow F \otimes \varphi \text{ is exact.}$$

According to [2, p. 45], F is called faithfully flat, if for every sequence φ,

$$\varphi \text{ is exact} \Longleftrightarrow F \otimes \varphi \text{ is exact.}$$

*Will be presented in English