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The existence of best proximity points for set-valued p-cyclic contractions

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Abstract

In this paper the concept of set-valued p-cyclic contraction map is introduced. The existence of best proximity point for such mappings on a metric space with the WUC property is presented.

Keywords: Best proximity point; Property WUC; Set-valued p-cyclic contraction map.
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1 Introduction

In 2003, Kirk et al. [6] established the following fixed point theorem.

Theorem 1.1. *[6] Let A and B be nonempty closed subsets of a complete metric space (X, d), and suppose $f : A \cup B \to A \cup B$ satisfies in the following condition:*

(i) $f(A) \subset B$ and $f(B) \subset A$.

(ii) $d(f(x), f(y)) \leq kd(x, y)$, $\forall x \in A$, $y \in B$, where $k \in (0, 1)$.

Then f has a unique fixed point in $A \cap B$.

Each map which satisfying in the assumption (i) of the above theorem is called cyclic map. Later on, Eldred and Veeramani [2] extended the contraction condition (ii) of the above theorem for cyclic maps as follows:

$$d(f(x), f(y)) \leq kd(x, y) + (1 - k)d(A, B),\ \forall x \in A\ \ y \in B,\ k \in (0, 1). \tag{1}$$

Every map which satisfying in (1) is said to be a cyclic contraction map. If f is a cyclic map on $A \cup B$, then a point $x \in A \cup B$ is called a best proximity point if $d(x, f(x)) = d(A, B)$, where

$$d(A, B) = \inf\{d(x, y) : x \in A, y \in B\}.$$

Eldred and Veeramani [2] studied cyclic contraction maps and obtained the existence of a best proximity point for cyclic contraction maps in metric spaces and uniformly convex Banach spaces.Then, in [7] the property UC occurs in a large collection of pairs of subsets

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