



A report of a project : Equivalence spaces

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Abstract

In this paper induced equivalence spaces and \mathcal{U} -products are introduced and discussed. also the notion of equivalently open subspace of a equivalence space and equivalently open functions are studied.

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Mathematics Subject Classification [2010]: 54H99

1 Introduction

In this paper we deal with equivalence spaces. An equivalence space is a structure close to uniform spaces (Uniform spaces are somewhere the mid way points between metric spaces on one hand and abstract topological spaces on the other hand). These spaces have been introduced first in 2014 by F. Omidi and M.R. molaei [1]. There are however a few aspects of metric spaces that are lost in general topological spaces. For example, since the notion of nearness is not defined for a general topological space, we cannot define the notion of uniform continuity in abstract topological spaces. The same can be said about notions such as total boundedness. An equivalence space is a mathematical construction in which such uniform concepts are still available.

An equivalence space (X, \mathcal{U}) is a set X along with a collection \mathcal{U} of equivalence relations on X such that \mathcal{U} is closed under finite intersections. We refer to \mathcal{U} as equivalence collection on X .

A function $f : X \rightarrow Y$ where (X, \mathcal{U}) and (Y, \mathcal{V}) are two equivalence spaces, is called equivalently continuous if $(f \times f)^{-1}(V) \in \mathcal{U}$ whenever $V \in \mathcal{V}$, where

$$(f \times f)^{-1}(V) = \{(x, y) \in X \times X \mid (f(x), f(y)) \in V\}.$$

Moreover, if (X, \mathcal{U}) is an equivalence space, then the collection $\mathcal{T}_{\mathcal{U}} = \{G \subseteq X \mid \text{for each } x \in G, \text{ there exists } U \in \mathcal{U} \text{ such that } U[x] \subseteq G\}$ is a topology on X with the base $\{U[x] \mid U \in \mathcal{U}, x \in X\}$ where $U[x] = \{y \in X \mid (x, y) \in U\}$. We refer to $\mathcal{T}_{\mathcal{U}}$ as the \mathcal{U} -induced topology.

We are going to consider induced equivalence spaces and \mathcal{U} -products. Also, we will introduce and discuss equivalently open subspaces.

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