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Causality conditions and cosmological time function

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Abstract

In this paper the concept of dual cosmological time function and its regularity is introduced. It is shown that the regularity of cosmological and dual cosmological time functions are independent of each other. It is proved that if the cosmological time function of the spacetime (M, g) is continuous, $\tau \to 0$ along every past inextendible causal curve and $\tau \to \infty$ along every future inextendible causal curve then (M, g) is globally hyperbolic.

Keywords: Spacetime, Globally hyperbolic, Cosmological time function, Lorentzian metric
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1 Introduction

In this paper we investigate the concept of cosmological time function and its relation with the causal hierarchy of the spacetime. So let us recall some ladders in the causal hierarchy which are needed in this paper.

Definition 1.1. [2] A spacetime is non-total future imprisoning if no future inextendible causal curve is totally future imprisoned in a compact set. A spacetime is non-partially future imprisoning if no future inextendible causal curve is partially future imprisoned in a compact set. Analogue definitions hold in the past case. A spacetime is non-total imprisoning if it is bout non total future and non total past imprisoning.

Definition 1.2. [2] A spacetime (M, g) is globally hyperbolic if it is causal and the intersections $J^+(p) \cap J^-(q)$ are compact for all $p, q \in M$.

The domain of dependence of A is defined as $D(A) = D^+(A) \cup D^-(A)$, where $D^+(A)$ (resp. $D^-(A)$) is defined as the set of points $p \in M$ such that every past (resp. future) inextendible causal curve through p intersects A.

Definition 1.3. [2] A Cauchy hypersurface is a subset $S \subset M$ which is crossed exactly once by any inextendible timelike curve ($D(S) = M$).

Equivalently it is proved that (M, g) is globally hyperbolic if it admits a Cauchy hypersurface [2].

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