



Second derivative general linear methods for the numerical solution of IVPs

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Abstract

General linear methods (GLMs) were introduced as the natural generalizations of the classical RungeKutta and linear multistep methods. An extension of GLMs, so-called SGLMs (GLM with second derivative), was introduced to the case in which second derivatives, as well as first derivatives, can be calculated. In this paper, we introduce the basic concepts, construction and implementation of SGLMs.

Keywords: Stiff IVPs, General linear methods, Second derivative methods, stability aspects, Variable stepsize implementation.

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1 Introduction

Traditional numerical methods for solving an initial value problem

$$\begin{aligned}y'(x) &= f(y(x)), & x \in [x_0, \bar{x}], \\ y(x_0) &= y_0,\end{aligned}\tag{1}$$

where $f : \mathbb{R}^m \rightarrow \mathbb{R}^m$ and m is the dimensionality of the system, generally fall into two main classes: linear multistep (multistage) and Runge–Kutta (multistage) methods. In 1966, Butcher [5] introduced general linear methods as a unifying framework for the traditional methods to study the properties of consistency, stability, and convergence and to formulate new methods with clear advantages over the these classes.

On the other hand, one of the main directions to construct methods with higher order and extensive stability region, is the using higher derivatives of the solutions, and some methods have been introduced that have good properties, especially for stiff problems. See [7, 8, 10]. Although the mentioned GLM includes linear multistep methods, Runge–Kutta and many other standard methods, but for the above reasons, it has be seemed that it be extended to the case in which second derivatives of solution, as well as first derivatives, can be calculated. These methods introduced by Butcher and Hojjati [6].

In this paper, we will review the basic concepts, types, construction and implementation issues of SGLMs.

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