Nonlinear Analysis 74 (2011) 4754–4770

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Global structure of Riemann solutions to a system of two-dimensional hyperbolic conservation laws☆

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ARTICLE INFO

Article history:
Received 12 May 2010
Accepted 22 April 2011
Communicated by Ravi Agarwal

MSC:
35L65
35L67
76N15

Keywords:
Two-dimensional conservation laws
Riemann problems
Global structure of solutions
Generalized characteristic analysis
Delta shock wave

ABSTRACT

The Riemann problem for a two-dimensional nonstrictly hyperbolic system of conservation laws is considered. Without the restriction that each jump of the initial data projects one planar elementary wave, ten topologically distinct solutions are obtained by applying the method of generalized characteristic analysis. Some of these solutions involve the nonclassical waves, i.e., the delta shock wave and the delta contact discontinuity, for which we explicitly give the expressions of their strengths, locations and propagation speeds. Moreover, we demonstrate that the nature of our solutions is identical with that of solutions to the corresponding one-dimensional Cauchy problem, which provides a verification that our construction produces the correct unique global solutions.

1. Introduction

The Riemann problem, a kind of Cauchy problem with the simplest discontinuous initial data, is the most fundamental problem in the field of nonlinear hyperbolic conservation laws. Compared to the Cauchy problem, it is much easier to study, but still reveals the basic properties of the Cauchy problem. Due to the explicit structure of the Riemann solutions, it also serves as a touchstone for numerical schemes.

For multi-dimensional systems of conservation laws, the well-known Euler system for compressible gases is the primary one. Many efforts have been devoted to the system, especially a body of work on the two-dimensional Riemann problem has been developed [1,2]. In [3], Zhang and Zheng investigated the four quadrant Riemann problem for the Euler equations, namely the initial data are four constant states in each quadrant of (x, y) plane. With the method of generalized characteristic analysis, they constructed the boundaries of interaction of four planar waves from infinity case by case. In the domains of interaction, they formulated a set of conjectures of the wave patterns. Although some efforts have been made to prove these conjectures in the past twenty years, it is unfortunate that none of them has been proved rigorously. The Euler system remains formidable for its complexity. This motivates our interest in considering simplified models which can capture various isolated features of the Euler system. In this paper, we focus our attention on the following system of conservation

☆ This work is partially supported by the National Natural Science Foundation of China (11001116, 10901077, 10871199) and China Postdoctoral Science Foundation (201003504, 20090451089).

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doi:10.1016/j.na.2011.04.044