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Inverse problem of groundwater modeling by iteratively regularized Gauss–Newton method with a nonlinear regularization term

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ABSTRACT

A nonlinear minimization problem $\|F(d) - u\| \longrightarrow \min$, $\|u - u_\delta\| \leq \delta$, is a typical mathematical model of various applied inverse problems. In order to solve this problem numerically in the lack of regularity, we introduce iteratively regularized Gauss–Newton procedure with a nonlinear regularization term (IRGN–NRT). The new algorithm combines two very powerful features: iterative regularization and the most general stabilizing term that can be updated at every step of the iterative process. The convergence analysis is carried out in the presence of noise in the data and in the modified source condition. Numerical simulations for a parameter identification ill-posed problem arising in groundwater modeling demonstrate the efficiency of the proposed method.

1. Introduction

The original iteratively regularized Gauss–Newton (IRGN) process [1,2]

$$d_{n+1} = d_n - \alpha_n[F'^*(d_n)F'(d_n) + \tau_n I]^{-1}(F'^*(d_n)(F(d_n) - u_\delta) + \tau_n(d_n - \xi)) \quad (1.1)$$

was introduced by Bakushinsky in 1993 for solving a nonlinear operator equation

$$F(d) = u, \qquad \|u - u_\delta\| \leq \delta, \quad (1.2)$$

or a more general problem of minimizing a nonlinear functional

$$\|F(d) - u\| \longrightarrow \min \quad (1.3)$$

with $F : \mathcal{D} \subset \mathcal{X} \to \mathcal{Y}$ acting between two Hilbert spaces. The idea to regularize Gauss–Newton algorithm iteratively proved to be extremely effective. Method (1.1) was successfully applied to a number of nonlinear ill-posed problems [3–6]. One of the remarkable features of this scheme is the lack of the requirement on d_0 to be rather close to the exact solution $\hat{d}$. The larger the norm of $d_0 - \hat{d}$, the larger the value of τ_0 must be used at the initial step. However, since $\tau_n \to 0$ as $n \to \infty$, one can still get a high quality of reconstruction that is consistent with the level of noise in the data. Besides, for iteration (1.1) to converge, the nonlinear operator F does not need to be monotone or have any other restrictions on its spectrum. In order to carry out the convergence analysis, Bakushinsky used the so-called source conditions

$$\xi - \hat{d} = F'^*(\hat{d})v, \quad v \in \mathcal{Y}, \ \|v\| \leq \varsigma, \quad (1.4)$$

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