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Numerical determination of the basin of attraction for exponentially asymptotically autonomous dynamical systems

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ABSTRACT

Numerical methods to determine the basin of attraction for autonomous equations focus on a bounded subset of the phase space. For non-autonomous systems, any relevant subset of the phase space, which now includes the time as one coordinate, is unbounded in the t-direction. Hence, a numerical method would have to use infinitely many points.

To overcome this problem, we introduce a transformation of the phase space. Restricting ourselves to exponentially asymptotically autonomous systems, we can map the infinite time interval to a finite, compact one. The basin of attraction of a solution becomes the basin of attraction of an exponentially stable equilibrium for an autonomous system. Now we are able to generalise numerical methods from the autonomous case. More precisely, we characterise a Lyapunov function as a solution of a suitable linear first-order partial differential equation and approximate it using radial basis functions.

1. Introduction

Non-autonomous differential equations play an important role in modelling many processes in biology, industry and economics. We study a general equation of the form $\dot{x} = f(t, x)$, where f is smooth and $x \in \mathbb{R}^n$. Important questions are concerned with regard to the stability of a special solution and its basin of attraction consisting of all solutions that approach the special solution.

The study of non-autonomous differential equations has become a major field within dynamical systems; in contrast to the autonomous case, attractivity, attractors and basins of attractions in **forward** and **backward** time (pullback attractors) are now independent and different notions, for references see e.g. [1] for the study of a non-autonomous Lotka–Volterra system in forward and backward time or [2]. In this paper, we only consider forward attractivity.

In the case of equilibria in autonomous differential equations, the basin of attraction can be determined using sublevel sets of Lyapunov functions. Methods for the construction of Lyapunov functions in autonomous systems include Zubov's equation [3] and linear programming [4]. A special Lyapunov function can be characterised as the solution of a first-order partial differential equation, and then be approximated using Meshless Collocation, in particular radial basis functions [5,6]. The method was extended to discrete [7], non-smooth [8] and time-periodic systems [9].

Turning back to non-autonomous systems, we can add the time as one additional variable. However, the basin of attraction of a solution is now an unbounded set, since the phase space includes the time as one coordinate. Hence, a numerical method would have to use infinitely many points.

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