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# Generalized Besicovitch spaces and applications to deterministic homogenization

Mamadou Sango [a], Nils Svanstedt [b,*], Jean Louis Woukeng [c,1]

[a] Department of Mathematics and Applied Mathematics, University of Pretoria, Pretoria 0002, South Africa

[b] Department of Mathematical Sciences, Göteborg University, SE-412 96 Göteborg, Sweden

[c] Department of Mathematics and Computer Science, University of Dschang, P.O. Box 67, Dschang, Cameroon

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ABSTRACT

The purpose of the present work is to introduce a framework which enables us to study nonlinear homogenization problems. The starting point is the theory of *algebras with mean value*. Very often in physics, from very simple experimental data, one gets complicated structure phenomena. These phenomena are represented by functions which are permanent in mean, but complicated in detail. In addition the functions are subject to the verification of a functional equation which in general is nonlinear. The problem is therefore to give an interpretation of these phenomena using functions having the following qualitative properties: they are functions that represent a phenomenon on a large scale, and which vary irregularly, undergoing nonperiodic oscillations on a fine scale. In this work we study the qualitative properties of spaces of such functions, which we call *generalized Besicovitch spaces*, and we prove general compactness results related to these spaces. We then apply these results in order to study some new homogenization problems. One important achievement of this work is the resolution of the generalized weakly almost periodic homogenization problem for a nonlinear pseudo-monotone parabolic-type operator. We also give the answer to the question raised by Frid and Silva in their paper [35] [H. Frid, J. Silva, Homogenization of nonlinear pde's in the Fourier–Stieltjes algebras, SIAM J. Math. Anal, 41 (4) (2009) 1589–1620] as regards whether there exist or do not exist ergodic algebras that are not subalgebras of the Fourier–Stieltjes algebra.



## 1. Introduction

The concepts of algebras with mean value [1] (algebras w.m.v., for short) and of homogenization algebras [2] ($H$-algebras, for short) were introduced to extend to the context of more general classes of oscillatory functions (such as almost periodic functions and others) the theory of periodic homogenization. This gives rise to the theory of deterministic/individual homogenization.

An algebra w.m.v. $A$ in $\mathbb{R}^N$ is defined as a Banach subalgebra of the algebra of bounded uniformly continuous real valued functions $\mathcal{B}(\mathbb{R}^N)$, which is translation invariant, and contains the constants and whose elements possess a mean value. Thus, it generalizes the concept of almost periodic functions. In this sense therefore, from the definition of an algebra w.m.v., one easily introduces generalized Besicovitch spaces $B^p_A$ associated with an algebra w.m.v. $A$. Moreover with each algebra w.m.v. $A$ there is associated a compact topological space $\Delta(A)$ (called its spectrum) such that every element of $A$ can be seen as an

* Corresponding author.
*E-mail addresses:* mamadou.sango@up.ac.za (M. Sango), nilss@chalmers.se, nilss@math.chalmers.se (N. Svanstedt), jwoukeng@yahoo.fr (J.L. Woukeng).
[1] Current address: Department of Mathematics and Applied Mathematics, University of Pretoria, Pretoria 0002, South Africa.

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