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Invariant densities and escape rates: Rigorous and computable approximations in the L^∞-norm

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ABSTRACT

In this article, we study piecewise linear discretization schemes for transfer operators (Perron–Frobenius operators) associated with interval maps. We show how these can be used to provide rigorous **pointwise** approximations for invariant densities of Markov interval maps. We also derive the order of convergence of the approximate invariant density to the real one in the L^∞-norm. The outcome of this paper complements recent results on the formulae of escape rates of open dynamical systems, (Keller and Liverani, 2009) [7]. In particular, the novelty of our work over previous results on BV and L^∞ approximations is that it provides a method for explicit computation of the approximation error.

1. Introduction

Although this article is about the approximation of invariant densities for interval maps, it is intimately related to what are commonly termed *open dynamical systems* or maps with 'holes' [1,2]. Open dynamical systems have become a very active area of research. In part, this is due to their connection to metastable dynamical systems [3,4] and their applications in earth and ocean sciences [5,6]. Corresponding to invariant measures for closed dynamics, in open dynamical systems, long-term statistics are described by a *conditionally invariant measure* and its related *escape rate*, measuring the mass lost from the system per unit time [1].

In their recent article [7], Keller and Liverani obtained precise escape rate formulae for Lasota–Yorke maps with holes shrinking to a single point. These formulae depend, *pointwise*, on the invariant density of the corresponding closed system. Unfortunately, explicit formulae of invariant densities for Lasota–Yorke maps are, in general, unavailable. Thus, to complement the result of [7], it is natural to consider numerical schemes that provide rigorous and computable pointwise approximations of invariant densities.

In the literature, rigorous approximation results are available in the L^1-norm [8–11], the L^∞-norm [12] and in the BV-norm, the space of functions of bounded variation, [13,14]. None of these methods are well suited to our problem. For example, L^1 approximations cannot provide the pointwise information necessary for application of the formulae of [7] (see

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