Nonlinear Analysis 74 (2011) 1164–1181

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Decay of energy for second-order boundary hemivariational inequalities with coercive damping☆

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ARTICLE INFO

Article history:
Received 7 December 2009
Accepted 24 September 2010

MSC:
35L85
35B40

Keywords:
Hemivariational inequality
Asymptotic behavior
Exponential decay of energy
Coercive damping

ABSTRACT

In this article we consider the asymptotic behavior of solutions to second-order evolution inclusions with the boundary multivalued term $u''(t) + A(t, u'(t)) + Bu(t) + \bar{\gamma}^*\partial J(t, \bar{\gamma}u'(t)) \ni 0$ and $u''(t) + A(t, u'(t)) + Bu(t) + \bar{\gamma}^*\partial J(t, \bar{\gamma}u(t)) \ni 0$, where A is a (possibly) nonlinear coercive and pseudomonotone operator, B is linear, continuous, symmetric and coercive, $\bar{\gamma}$ is the trace operator and J is a locally Lipschitz integral functional with ∂ denoting the Clarke generalized gradient taken with respect to the second variable. For both cases we provide conditions under which the appropriately defined energy decays exponentially to zero as time tends to infinity. We discuss assumptions and provide examples of multivalued laws that satisfy them.

1. Introduction

Differential inclusions with the nonmonotone multivalued term given in the form of the Clarke subdifferential are known as hemivariational inequalities (HVIs). They are applied mainly to describe contact problems in the theory of (visco)elasticity where the multivalued and nonmonotone terms are used to represent various laws of normal contact, friction, adhesion, damage, or wear.

It is well known that under appropriate conditions the solutions of the quasilinear hyperbolic PDE of second-order $u'' + Au' + Bu = 0$ and their time derivatives tend to zero in an appropriate sense as the time tends to infinity. This behavior is called the energy decay. In the case when the operator A is coercive (the prototype of such an operator is $Au' = -\Delta u'$) we speak about the structural damping, and energy decay results are relatively easy to obtain. The behavior of the model in such a case is parabolic. More interest in the last 40 years was devoted to the case of pure viscous damping (the prototype of a viscous damping operator is $Au' = u'$) where the decay results are harder to obtain. It is well known that the type of decay (e.g. exponential, polynomial, logarithmic) and its rates are associated with the types of growth condition on the damping term at the origin and at infinity. In particular, the simplest case of linear growth both at the origin and at infinity leads to the exponential decay of energy. One method for proving such decay was developed by Nakao whose results concern the decay rates for the single-valued problems (see for instance [1,2]).

The problem with multivalued and maximal monotone viscous damping defined in the problem domain was studied by Haraux [3]. Later, Baji et al. [4] analyzed the problem governed by the inclusion $u'' + \partial\Phi(u') - \Delta u \ni 0$ with a continuous, convex function Φ such that $\mathrm{int}\,\partial\Phi(0) \neq \emptyset$ and established conditions under which the velocity decays to zero in finite time.

☆ Supported in part by the Ministry of Science and Higher Education of Poland under grant No. N201 027 32/1449.
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doi:10.1016/j.na.2010.09.053