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Global strong solutions of Navier–Stokes equations with interface boundary in three-dimensional thin domains

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ABSTRACT

In this article, we study the spectrum of the Stokes operator in a 3D two layer domain with interface, obtain the asymptotic estimates on the spectrum of the Stokes operator as thickness ε goes to zero. Based on the spectral decomposition of the Stokes operator, a new average-like operator is introduced and applied to the study of Navier–Stokes equation in the two layer thin domains under interface boundary condition. We prove the global existence of strong solutions to the 3D Navier–Stokes equations when the initial data and external forces are in large sets as the thickness of the domain is small. This article is a continuation of our study on the Stokes operator under Navier friction boundary condition. Due to the viscosity distinction between the two layers, the Stokes operator displays radically different spectral structure from that under Navier friction boundary condition, then causes great difficulty to the analysis.

1. Introduction

In this article, we study the global-in-time existence of the strong solutions to the 3D Navier–Stokes equations in thin domains with *interface boundary condition* in the vertical direction. We adopt the terminology of [1], the boundary condition of our interest should be denoted by IP, i.e. interface boundary condition in the vertical direction, while periodic boundary conditions in the horizontal directions. The interface boundary condition represents the interaction between two fluids, one is dragged by another. Mathematically, the IP states that the shear stress on the boundary is proportional to the difference of the horizontal velocities in each layer. Ideally, we assume the interface is immiscible. From the physical point of view, our study is motivated by the work [2–7] in the context of geophysical fluid dynamics, where the upper layer corresponds to the atmosphere and the lower layer is the ocean. From the mathematical point of view, our work is motivated by the work in [8,9] for general contact parabolic equations, in [10] concerning Navier–Stokes equations, where the global strong solutions to the Navier–Stokes equations in thin two layer domains have been studied and are proved to exist global-in-time for a large set of initial data and forcing terms. In spirit, this is a continuation of our work on Navier–Stokes equations under Navier friction boundary conditions, see [11], where the global existence of strong solution of Navier–Stokes equations were obtained by analyzing the spectrum of the Stokes operator.

The question of a solution of the three-dimensional incompressible Navier–Stokes equations can develop a finite time singularity from smooth initial data with finite energy is one of the most outstanding open problems in mathematics. Since its being formulated in [12], much endeavor has been devoted to tackling this open problem by assuming additional

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