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Galerkin and subspace decomposition methods in space and time for the Navier–Stokes equations☆

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ABSTRACT

The Galerkin method and the subspace decomposition method in space and time for the two-dimensional incompressible Navier–Stokes equations with the H^2-initial data are considered. The subspace decomposition method consists of splitting the approximate solution as the sum of a low frequency component discretized by the small time step Δt and a high frequency one discretized by the large time step $p\Delta t$ with $p > 1$. The H^2-stability and L^2-error analysis for the subspace decomposition method are obtained. Finally, some numerical tests to confirm the theoretical results are provided.

1. Introduction

Subspace decomposition methods (the multi-level methods) are based on the concepts of inertial manifolds and approximate inertial manifolds (see [1–5]); see, e.g., [6–8] for the Navier–Stokes equations and Burie and Marion [9] for parabolic problems and Marion and Mollard [10] for the convection–diffusion problems. These methods are based on a two-level spatial decomposition and the use of different time steps for the various spatial components.

Recently, in contrast to the above subspace decomposition methods, another multi-level methods based on multi-level spatial decomposition and one or two Newton correction steps are developed, for example, see [11–13] for the multi-level finite element method for the stationary Navier–Stokes equations, He and Liu [14] for the multi-level finite element method for the time-dependent Navier–Stokes equations, and He et al. [15] and He and Liu [16] for the multi-level spectral Galerkin method for the time-dependent Navier–Stokes equations. Moreover, many authors have studied some post-processing Galerkin (PPG) methods related to the subspace decomposition method, (see, e.g., [17–23]) for the case of the body forcing f being time-independent or Hölder continuous and the reform PPG method or the dynamic PPG method, and (see, e.g., [24,22]) for the case of the body forcing f being time-dependent. However, to obtain the low frequency component of the numerical solution, the multi-level spectral methods and the post-processing Galerkin methods need to solve some full nonlinear Navier–Stokes equations on the low frequency subspace.

In order to avoid to solving the full nonlinear Navier–Stokes equations on the low frequency subspace, in this paper we consider the standard Galerkin and subspace decomposition methods for the time-dependent Navier–Stokes equations with

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