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Intersecting nonhomogeneous Cantor sets with their translations

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ABSTRACT

A scheme is given to compute the Hausdorff dimensions for the intersection of a class of nonhomogeneous Cantor sets with their translations.

1. Introduction

Let $\beta \in (0, 1/2)$ and let

$$\psi_k(x) = \beta x + k(1-\beta), \quad k = -1, 0, 1.$$

The middle-$(1-2\beta)$ Cantor set $\Gamma_\beta \subseteq [0, 1]$ is defined as the unique invariant nonempty compact set under maps ψ_0 and ψ_1:

$$\Gamma_\beta = \psi_0(\Gamma_\beta) \cup \psi_1(\Gamma_\beta). \tag{1}$$

One also call Γ_β, the self-similar set generated by the iterated function system (IFS) $\{\psi_0, \psi_1\}$. The set $\Gamma_{1/3}$ is the classical middle third Cantor set. In the past two decades, intersection of Cantor sets has been the subject of several studies [1–12]. The context and motivation are numerous, but mainly come from the discipline of dynamical systems. A brief history of why intersection of Cantor sets is important in dynamical systems is described by Davis and Hu in [1]. The intersection $\Gamma_{1/3} \cap (\Gamma_{1/3} + t)$ (or more general, $\Gamma_\beta \cap (\Gamma_\beta + t)$) has been extensively studied by lots of authors.

When $0 < \beta < 1/3$, $\Gamma_\beta - \Gamma_\beta$ is the self-similar set generated by the IFS $\{\psi_0, \psi_1, \psi_{-1}\}$ and satisfies the open set condition (see (3) in Section 2) so that for each $t \in \Gamma_\beta - \Gamma_\beta$ the set $\Gamma_\beta \cap (\Gamma_\beta + t)$ is just a generalized Moran set (see (5) in Section 2). Thus, the Hausdorff, packing and box-counting dimensions of $\Gamma_\beta \cap (\Gamma_\beta + t)$ can then be determined. When $\beta = 1/3$, it can be dealt with in the same way though a bit of difficulty occurs. However, when $1/3 < \beta < 1/2$, the set $\Gamma_\beta \cap (\Gamma_\beta + t)$ presents very complicated geometrical structure (see (4) in Section 2). A natural question is the following.

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