Nonlinear Analysis 74 (2011) 3531–3539

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Monotone iterative sequences for nonlinear boundary value problems of fractional order

Mohammed Al-Refai [*,1], Mohamed Ali Hajji

Department of Mathematical Sciences, United Arab Emirates University, P.O. Box 17551, Al Ain, United Arab Emirates

A R T I C L E I N F O

Article history:
Received 1 November 2010
Accepted 3 March 2011
Accepting Editor: Ravi Agarwal

MSC:
34A08
34Bxx
35J40

Keywords:
Fractional differential equations
Boundary value problems
Maximum principle
Lower and upper solutions
Caputo fractional derivative

A B S T R A C T

In this paper we extend the maximum principle and the method of upper and lower solutions to boundary value problems with the Caputo fractional derivative. We establish positivity and uniqueness results for the problem. We then introduce two well-defined monotone sequences of upper and lower solutions which converge uniformly to the actual solution of the problem. A numerical iterative scheme is introduced to obtain an accurate approximate solution for the problem. The accuracy and efficiency of the new approach are tested through two examples.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

We consider the boundary value problem of fractional order

$$D^{\delta}u(t)+g(t,u)=0,\quad t\in(0,1),\ 1<\delta\le 2, \tag{1.1}$$

$$u(0)=a,\qquad u(1)=b, \tag{1.2}$$

where $g:[0,1]\times\mathbb{R}\to\mathbb{R}$ is a continuous function, $a,b\in\mathbb{R}$, and D^{δ} is the Caputo fractional derivative of order δ. For $\delta>0,\ m-1<\delta\le m,\ m\in\mathbb{N}$, the Caputo derivative (see [1,2]) is defined by

$$D^{\delta}f(t)=\frac{1}{\Gamma(m-\delta)}\int_0^t (t-s)^{m-1-\delta}f^{(m)}(s)\mathrm{d}s. \tag{1.3}$$

The Caputo derivative defined in (1.3) is related the Riemann–Liouville fractional integral, I^{δ}, of order $\delta\in\mathbb{R}^{+}$, by

$$D^{\delta}f(t)=I^{m-\delta}f^{(m)}(t), \tag{1.4}$$

where for any $\alpha>0$, (see [2,3])

$$I^{\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}f(s)\mathrm{d}s. \tag{1.5}$$

* Corresponding author. Tel.: +971 50 1278390; fax: +971 3 7671291.
E-mail addresses: m_alrefai@uaeu.ac.ae (M. Al-Refai), mahajji@uaeu.ac.ae (M. Ali Hajji).
1 On leave from Department of Mathematics and Statistics, Jordan University of Science and Technology, P.O. Box 3030, Irbid, 22110, Jordan.

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2011.03.006