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Boundary controllability for the quasi-linear wave equations coupled in parallel

Li Deng [a,*], Bopeng Rao [b], Peng-Fei Yao [c]

[a] Institute of Mathematics, Southwest Jiaotong University, Chengdu 610031, PR China

[b] Institut de Recherche Mathematique Avancee, Universite Louis Pasteur de Strasbourg, 7 rue Rene Descartes 67084 Strasbourg Cedex, France

[c] Key Laboratory of Control and Systems, Institute of Systems Science, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, PR China

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ABSTRACT

We study the boundary exact controllability for a system of two quasi-linear wave equations coupled in parallel with springs and viscous terms. We prove the locally exact controllability around superposition equilibria under some checkable geometrical conditions. We then establish the globally exact controllability in such a way that the state of the coupled quasi-linear system moves from a superposition equilibrium in one location to a superposition equilibrium in another location. Our results show that exact controllability is geometrical characters of a Riemannian metric, given by the coefficients and superposition equilibria of the system.

1. Introduction and main results

The control of vibration for an elastic object has since the 1970's been thoroughly investigated. Linear wave equations describe the vibration of elastic objects with simplifying assumptions. The results of controllability for linear wave equations date back to [1–9], and so on. In the case of one dimensional quasi-linear wave equations, we refer the readers to [10–12], etc., and in the case of higher dimensions, we refer the readers to [13,14].

Controllability for coupled linear wave equations has been investigated by many authors, we refer the readers to [15–20], etc. There are few results in higher dimensions for nonlinear cases.

We consider boundary controllability for a system coupled by two quasi-linear wave equations in parallel. We state our problem as follows.

Let $n \geq 2$ be an integer. Let $\Omega \subset \mathbb{R}^n$ be a bounded, open set with smooth boundary $\Gamma = \Gamma_0 \cup \Gamma_1$ and $\Gamma_0 \cap \Gamma_1 = \emptyset$. Let ν be the outward unit normal under the dot metric. Let

$$\mathbf{a}(x, y) = (a_1(x, y), \dots, a_n(x, y)) : \overline{\Omega} \times \mathbb{R}^n \to \mathbb{R}^n$$

be a smooth mapping with

$$\mathbf{a}(x, 0) = 0 \quad \text{for } x \in \overline{\Omega}, \tag{1.1}$$

such that $(a_{ij}(x, y))$ is symmetric and

$$A(x, y) = (a_{ij}(x, y)) > 0, \quad (x, y) \in \overline{\Omega} \times \mathbb{R}^n, \tag{1.2}$$

where $a_{ij} = a_{iy_j}$ are the partial derivatives of a_i with respect to the variable y_j.

* Corresponding author. Tel.: +86 18981876776.
E-mail addresses: dengli@home.swjtu.edu.cn (L. Deng), pfyao@iss.ac.cn (P.-F. Yao).

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