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Compositions and averages of two resolvents: Relative geometry of fixed points sets and a partial answer to a question by C. Byrne

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ABSTRACT

We show that the set of fixed points of the average of two resolvents can be found from the set of fixed points for compositions of two resolvents associated with scaled monotone operators. Recently, the proximal average has attracted considerable attention in convex analysis. Our results imply that the minimizers of proximal-average functions can be found from the set of fixed points for compositions of two proximal mappings associated with scaled convex functions. When both convex functions in the proximal average are indicator functions of convex sets, least squares solutions can be completely recovered from the limiting cycles given by compositions of two projection mappings. This provides a partial answer to a question posed by C. Byrne. A novelty of our approach is to use the notion of resolvent average and proximal average.

1. Introduction

Throughout, $\mathcal{H}$ is a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and induced norm $\|\cdot\|$, and $\Gamma(\mathcal{H})$ is the set of proper lower semicontinuous convex functions on $\mathcal{H}$. Let $A:\mathcal{H}\rightrightarrows\mathcal{H}$ be a set-valued operator with graph $\operatorname{gr}A := \big\{(x,u)\in\mathcal{H}\times\mathcal{H} \mid u\in Ax\big\}$. The set-valued inverse A^{-1} of A has graph $\big\{(u,x)\in\mathcal{H} \mid u\in Ax\big\}$, and the resolvent of A is $J_A := (A+\mathrm{Id})^{-1}$ where $\mathrm{Id}:\mathcal{H}\to\mathcal{H}$ denotes the identity mapping. The operator A is monotone if $\langle x-y, u-v\rangle \geq 0$ for all $(x,u),(y,v)\in\operatorname{gr}A$; A is maximal monotone if A is monotone and no proper enlargement of $\operatorname{gr}A$ is monotone.

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