Harnack estimates for a quasi-linear parabolic equation with a singular weight[☆]

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ABSTRACT

In this paper, based on measure theoretical arguments, we establish Harnack estimates and Hölder continuity of nonnegative weak solutions for a degenerate parabolic equation with a singular weight. We transform the equation by performing the change of function. The energy estimates, the upper boundedness, the lower boundedness and the expansion of positivity for the solutions to the transformed equation are obtained. Then our aim is reached.

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1. Introduction

In this paper, we are concerned with the quasi-linear parabolic equation with a singular weight

$$\partial_t u - \operatorname{div} A(x, t, u^m, \nabla u^m) = B(x, t, u^m, \nabla u^m), \quad (x, t) \in \Omega_T, \tag{1.1}$$

where $\Omega_T \equiv \Omega \times (0, T)$, Ω is a bounded domain in R^N containing the origin O, $0 < T < +\infty$, the functions $A : \Omega_T \times R^{N+1} \to R^N$ and $B : \Omega_T \times R^{N+1} \to R$ are assumed to be measurable and satisfy the structure conditions

$$A(x, t, u^m, \nabla u^m) \cdot \nabla u^m \geq |x|^{p\gamma} \{C_0 |\nabla u^m|^p - C_1^p\}, \tag{1.2}$$

$$|A(x, t, u^m, \nabla u^m)| \leq |x|^{p\gamma} \{C_2 |\nabla u^m|^{p-1} + C_1^{p-1}\}, \tag{1.3}$$

$$|B(x, t, u^m, \nabla u^m)| \leq |x|^{p\gamma} \{C_1 |\nabla u^m|^{p-1} + C_1^{p-1}\}, \tag{1.4}$$

where $2 < p < N$, $1 - \frac{N}{p} \leq \gamma \leq 0$, $m \geq 1$, C_0, C_2 are positive constants and C_1 is a nonnegative constant.

It is well known that (1.1) is an important doubly degenerate diffusion equation. The physical phenomena described by (1.1) can be seen in [1].

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