Nonlinear Analysis 74 (2011) 1272–1278

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Nonlinear discrete boundary value problems for the discrete p-Laplacian with potential term

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ARTICLE INFO

Article history:
Received 3 August 2010
Accepted 30 September 2010

Keywords:
Discrete boundary value problems
Discrete p-Laplacian
Graph Laplacian

ABSTRACT

In this paper, we show the existence of a positive solution of nonlinear discrete boundary value problems for the discrete p-Laplacian with potential term. Moreover, the multiplicity of solutions for the problems is also investigated.

1. Introduction

The theory of difference equations is rich in applications in many branches of the natural sciences, such as computer science, food webs, mechanical engineering, control systems, artificial or biological neural networks, global economics and many others, for instance, [1–5]. In particular, many researchers have taken an active interest in various discrete problems involving the discrete p-Laplacian, for instance, [6–16].

The purpose of this paper is to investigate the existence of a positive solution of the following discrete boundary value problems:

$$\begin{cases} -\Delta_{p,\omega} u(x) = -V(x)|u(x)|^{q-2}u(x) + f(x, u(x)), & x \in S, \\ u|_{\partial S} = 0 \end{cases} \tag{1}$$

where $p > 1, q > 1$, $\bar{S}$ is a finite, simple, connected, undirected and weighted graph composed of boundary ∂S and interior S with a weight ω which is a function $\omega : \bar{S} \times \bar{S} \to [0, \infty)$ satisfying

(i) $\omega(x, y) = \omega(y, x) > 0$ if $x \sim y$,

(ii) $\omega(x, y) = 0$ if and only if $\{x, y\} \notin E$.

(Here, as conventionally used, we denote by $x \in \bar{S}$ the fact that x is a vertex in a graph $\bar{S}$ and a set of edges in $\bar{S}$ is denoted by E.) V is a real valued function defined on the vertices set of a graph S, $f \in C(S \times (0, \infty))$ is a real valued function and

$$\Delta_{p,\omega} u(x) := \sum_{y \in \bar{S}} |u(y) - u(x)|^{p-2}(u(y) - u(x))\omega(x, y), \quad x \in S,$$

is a *discrete p-Laplacian* (or *graph p-Laplacian*). Here, the given function f may be singular at $(x, 0)$ for some $x \in S$.

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0362-546X/$ – see front matter
doi:10.1016/j.na.2010.09.065