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Holomorphic extension of solutions of semilinear elliptic equations

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ABSTRACT

We prove sharp analytic estimates and holomorphic extensions in sectors of $\mathbb{C}^d$ for the solutions of a wide class of semilinear elliptic differential and pseudodifferential equations in $\mathbb{R}^d$, improving earlier results where the extension was shown for a strip. Moreover, we derive exponential decay estimates for such extended solutions. The results presented apply in particular to solitary wave solutions of many classical nonlinear evolution equations as KdV-type, long-wave-type and Schrödinger equations.

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1. Introduction

The main concern of this paper is the study of holomorphic extensions of the solutions of semilinear elliptic equations in $\mathbb{R}^d$. Broadly speaking, we deal with equations of the form

$$Pu = F[u], \tag{1.1}$$

where P is a linear elliptic differential, or even pseudodifferential, operator in $\mathbb{R}^d$ and $F[u]$ is a nonlinearity, possibly involving the derivatives of u. For a wide class of equations of this type, it is known that every solution u, sufficiently regular and with a certain decay at infinity, actually is analytic on $\mathbb{R}^d$ and it extends to a holomorphic function in a strip of $\mathbb{C}^d$ of the form

$$\{z = x + \mathrm{i}y \in \mathbb{C}^d : \ |y| < \varepsilon\}, \tag{1.2}$$

for some $\varepsilon > 0$, satisfying there the estimates

$$|u(x + \mathrm{i}y)| \leq C\mathrm{e}^{-c|x|} \tag{1.3}$$

for some $C > 0$, $c > 0$. A pioneering work on this subject was the paper by Kato and Masuda [1]. Later the problem of the holomorphic extension in a strip has been intensively studied in connection with the applications to solitary wave equations. In particular, it was noticed in dimension 1 that several model equations like the Korteweg–de Vries equations and its generalizations, Schrödinger-type equations and long-wave-type equations admit solitary wave solutions which extend to meromorphic functions with poles out of a strip of the form (1.2) and having a decay of type (1.3). Among the

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