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Quasi-periodic waves of the $\mathcal{N} = 1$ supersymmetric modified Korteweg–de Vries equation

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ABSTRACT

Based on the Hirota bilinear method and the Riemann theta function, a straightforward way is shown to construct quasi-periodic wave solutions of supersymmetric equations. The resulting theory is applied to the supersymmetric modified Korteweg–de Vries equation. Further, we analyze the asymptotic properties of the solutions and give their asymptotic relations between the quasi-periodic wave solutions and the soliton solutions.

1. Introduction

The concept of supersymmetry was originally introduced and developed for applications in elementary particle physics 30 years ago [1–3]. It is also found that supersymmetry can be applied to a variety of problems such as relativistic and non-relativistic physics and nuclear physics. The mathematical formulation of supersymmetry is based on the introduction of Grassmann variables [4]. In recent years, supersymmetry has been a subject of considerable interest in physics and mathematics. A number of well-known mathematical physical equations have been generalized into supersymmetric analogues, such as supersymmetric versions of the sine–Gordon, Korteweg–de Vries (KdV), KP hierarchy, Boussinesq, and modified Korteweg–de Vries (MKdV) equations. It has been shown that these supersymmetric integrable systems possess bi-Hamiltonian structure, Painleve property, infinitely many symmetries, Darboux transformation, Backlund transformation, bilinear form and multi-soliton solutions [5–11]. The systematic bilinear transcription of supersymmetric equations was introduced by Carstea [11]; it requires an extension of the Hirota bilinear operator [12–17] to supersymmetric case. In recent years, Carsta, Liu, Ghosh et al. have done a lot of work on the construction of soliton solutions of supersymmetric equations [7–11,18]. However, to the best of our knowledge, the quasi-periodic solutions of the supersymmetric systems, which can be considered as a generalization of the soliton solutions, have been considered rarely so far.

The motivation of this paper is to show how the quasi-periodic wave solutions of nonlinear supersymmetric equations can be constructed with Hirota's bilinear method in superspace. To arrive at this aim, we devise a Riemannn theta function formula, which actually provides us with a lucid and straightforward method to apply to nonlinear supersymmetric equations. Once a nonlinear equation is written in bilinear form, the quasi-periodic wave solutions of the nonlinear equation

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