Property β of Rolewicz and orthogonal convexities of Calderón–Lozanovskiĭ spaces

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A R T I C L E   I N F O

A B S T R A C T

We find criteria for property β of Rolewicz and orthogonal convexities in Calderón–Lozanovskiĭ spaces. In order to present characterization for property β in the sequence case, we introduce a new respective monotonicity property. The particular case of Orlicz–Lorentz spaces will also be discussed.

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1. Introduction

The metric geometry of Banach spaces has been intensively developed during the last decades. Although some topics deal with general Banach spaces, many authors often consider different special classes of spaces such as Köthe–Bochner spaces, Banach lattices, Calderón–Lozanovskiĭ and Orlicz–Lorentz spaces. In the metric geometry of Banach lattices, it is natural to study monotonicity and rotundity properties, which are closely related (see [1]). Let us consider these topics in Calderón–Lozanovskiĭ spaces E_φ. In regard to monotonicities, the situation is as follows: E_φ has the monotonicity property M iff E has the same property M and φ is appropriately monotone (see [2–5]). The study of rotundity properties is more complicated: E_φ has the rotundity property R iff E has the respective monotonicity property M, φ is appropriately monotone and either E has the rotundity property R or φ is suitably convex. This idea was first applied in [5] and it was followed in several papers. We shall also use it here for the property β of Rolewicz and orthogonal uniform convexity. Note that these properties are connected with the fixed point property.

2. Preliminaries

Throughout this paper, X is a real Banach space. As usual, $S(X)$ and $B(X)$ stand for the unit sphere and the closed unit ball of X, respectively.

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