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Variational sets: Calculus and applications to nonsmooth vector optimization

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ABSTRACT

We develop elements of calculus of variational sets for set-valued mappings, which were recently introduced in Khanh and Tuan (2008) [1,2] to replace generalized derivatives in establishing optimality conditions in nonsmooth optimization. Most of the usual calculus rules, from chain and sum rules to rules for unions, intersections, products and other operations on mappings, are established. Direct applications in stability and optimality conditions for various vector optimization problems are provided.

1. Introduction

In nonsmooth optimization, many generalized derivatives have been introduced to replace the Fréchet and Gateaux derivatives which do not exist. Each of them is adequate for some classes of problems, but not all. In [1,2] we proposed two kinds of variational sets for mappings between normed spaces. These subsets of the image space are larger than the images of the pre-image space through known generalized set-valued mappings. Hence our necessary optimality conditions obtained by separation techniques are stronger than many known conditions using various generalized derivatives. Of course, sufficient optimality conditions based on separations of bigger sets may be weaker. But in [1,2], using variational sets we can establish sufficient conditions which have almost no gap with the corresponding necessary ones. The second advantage of the variational sets is that we can define these sets of any order to get higher-order optimality conditions. This feature is significant since many important and powerful generalized derivatives can be defined only for the first and second orders and the higher-order optimality conditions available in the literature are much fewer than the first and second-order ones. The third strong point of the variational sets is that almost no assumptions are needed to be imposed for their being well-defined and nonempty and also for establishing optimality conditions. Calculating them from the definition is only a computation of a Painlevé–Kuratowski limit. However, in [1,2] no calculus rules for variational sets are provided.

In the present paper we establish elements of calculus for variational sets to ensure that they can be used in practice. Most of the usual rules, from the sum and chain rules to various operations in analysis, are investigated. It turns out that the

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