An integral representation approach for valuing American-style installment options with continuous payment plan

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ABSTRACT

In this paper, we present an integral equation approach for the valuation of American-style installment derivatives when the payment plan is assumed to be a continuous function of the asset price and time. The contribution of this study is threefold. First, we show that in the Black–Scholes model the option pricing problem can be formulated as a free boundary problem under very general conditions on payoff structure and payment schedule. Second, by applying a Fourier transform-based solution technique, we derive a system of coupled recursive integral equations for the pair of free boundaries along with an analytic representation of the option price. Third, based on these results, we propose a unified framework which generalizes the existing methods and is capable of dealing with a wide range of monotonic payoff functions and continuous payment plans. Finally, by using the illustrative example of American vanilla installment call options, an explicit pricing formula is obtained for time-varying payment schedules.

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1. Introduction

Installment option contracts are exotic derivatives in which the premium (or price) is paid in installments of a payment plan spread over the lifetime of the option, rather than as a lump-sum at the time of purchase, and where it can be allowed to lapse the contract at any payment date before maturity. Option holder's right to drop the contract can be viewed as an early exercise feature and leads to free boundary problems similar to that arising for standard American options. An installment option with payments at pre-determined time intervals (usually monthly, quarterly or annually) is referred to as a *discrete-installment* (DI) option, whereas its continuous-time limit in which premium is paid at a certain rate per unit time is referred to as a *continuous-installment* (CI) option. In this paper, we consider the class of American-style CI options in which the buyer pays a small up-front premium and then installments of a payment plan, which is assumed to be a continuous function of the asset price and time, to acquire and keep the right, but not the obligation, to buy or sell the underlying asset on or before the maturity date. However, the holder can choose at any time to terminate installment payments, in which case the option lapses with no further payments on either side.

There are relatively few and quite recent studies on installment options. The earliest article on standard installment options seems to be [1], while more recently [2,3] obtained no-arbitrage bounds on the initial premium of European DI options, which were used to compare static and dynamic hedging strategies. [4] developed a dynamic-programming procedure to price American DI options and investigated the properties of installment options through theoretical and numerical analysis. Extending the concept of compound options, [5] derived a closed-form solution to the initial premium of European DI options and examined the limiting case of installment options with continuous payment plan. [6,7] used a partial Laplace transform to solve the free boundary problem arising from the pricing of European CI options and obtained asymptotic properties of the optimal stopping boundary close to expiry. For American and European CI options, [8,9]

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