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Global compactness results for quasilinear elliptic problems with combined critical Sobolev–Hardy terms☆

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ABSTRACT

In this paper, we study the global compactness results for quasilinear elliptic problems involving combined critical Sobolev–Hardy terms on the whole space and a bounded smooth domain, respectively. That is, we give the complete descriptions for the Palais–Smale (PS) sequences of the corresponding energy functionals. By using these descriptions, the existence results of solutions are also obtained.

1. Introduction

We consider the following quasilinear elliptic problem:

$$\begin{cases} -\Delta_p u = \mu \dfrac{|u|^{p-2}u}{|x|^p} + K(x)\dfrac{|u|^{p^*(s)-2}u}{|x|^s} + Q(x)\dfrac{|u|^{p^*(t)-2}u}{|x-x_0|^t} + f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a smooth domain in $\mathbb{R}^N$ $(N \geq 3)$, bounded or not, $-\Delta_p u = -\mathrm{div}\left(|\nabla u|^{p-2}\nabla u\right)$, $1 < p < N$, $0 \leq \mu < \overline{\mu} := \left(\frac{N-p}{p}\right)^p$, $x_0 \neq 0$ and $0, x_0 \in \Omega$, $0 < s \leq t < p$. $K(x)$ and $Q(x)$ are two nonnegative continuous functions. When $\Omega = \mathbb{R}^N$, the boundary condition $u = 0$ on $\partial\Omega$ means $u(x) \to 0$ as $|x| \to \infty$. $p^*(s) := \frac{p(N-s)}{N-p}$ and $p^*(t) := \frac{p(N-t)}{N-p}$ are critical Sobolev–Hardy exponents. Note that $p^*(0) = p^* := \frac{Np}{N-p}$ is the critical Sobolev exponent. We point out that all of the above are assumed throughout this paper except if otherwise specified.

In recent years, much attention has been paid to the existence of nontrivial solutions for the quasilinear elliptic problem (1.1). It is well known that the nontrivial weak solutions for (1.1) are equivalent to the nonzero critical points of the energy

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