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Compact operators into the spaces of strongly C_1 summable and bounded sequences

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ABSTRACT

We establish the necessary and sufficient conditions for the entries of an infinite matrix to map any of the classical sequence spaces $\ell_p (1 \leq p \leq \infty)$, c_0 and c into the spaces w_0, w and w_∞ of all sequences that are strongly summable to zero, strongly summable and strongly bounded, by the Cesàro method of order 1. We also give the representations of the general bounded linear operators from c into any of the spaces w_0, w and w_∞, and compute or estimate the Hausdorff measure of noncompactness in each case. Finally, we apply our results to characterise the compact linear operators between the spaces mentioned above, and generalise two classical results by Steinhaus (1911) [5], Maddox (1967) [31], and Cohen and Dunford (1937) [24].

1. Introduction, notations and known results

The spaces of all sequences that are strongly summable to zero, strongly summable, and strongly bounded, with index $p > 0$, were first introduced by Maddox [1] who determined their dual spaces and characterised the class of matrix transformations from the space of all sequences that are strongly summable into the space of all convergent series. A complete list was given in [2] of the characterisations of all matrix transformations from Maddox's spaces into the classical spaces of bounded, convergent, null sequences, and absolutely convergent series.

Here we establish the complementary results of the characterisations of all matrix transformations from the classical sequence spaces, including the space of all p summable sequences, into Maddox's spaces with index $p = 1$. We also extend our studies from the normally considered matrix transformations to general bounded linear operators between those spaces, except, of course, in the case of the initial space being the space of bounded sequences. Furthermore, we characterise the corresponding classes of all compact bounded linear operators and of all matrix operators, the single exception being the case of compact bounded linear operators between the spaces of convergent and strongly bounded sequences. The characterisation of this class remains an open problem which seems to be not solvable by our methods.

As usual, let ω be the set of all complex sequences $x = (x_k)_{k=1}^{\infty}$. By ℓ_∞, c, c_0 and ϕ, we denote the sets of all bounded, convergent, null and finite sequences, respectively; we also write $\ell_p = \left\{x \in \omega : \sum_{k=1}^{\infty} |x_k|^p < \infty\right\}$ for $1 \leq p < \infty$, and bs and cs for the sets of all bounded and convergent series.

Let e and $e^{(n)}$ $(n = 1, 2, \ldots)$ be the sequences with $e_k = 1$ for all k, and $e_n^{(n)} = 1$ and $e_k^{(n)} = 0$ $(k \neq n)$.

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