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Resonance and rotation numbers for planar Hamiltonian systems: Multiplicity results via the Poincaré–Birkhoff theorem

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ABSTRACT

In the general setting of a planar first order system

$$u' = G(t, u), \quad u \in \mathbb{R}^2, \tag{0.1}$$

with $G : [0, T] \times \mathbb{R}^2 \to \mathbb{R}^2$, we study the relationships between some classical nonresonance conditions (including the Landesman–Lazer one) — at infinity and, in the unforced case, i.e. $G(t, 0) \equiv 0$, at zero — and the rotation numbers of "large" and "small" solutions of (0.1), respectively. Such estimates are then used to establish, via the Poincaré–Birkhoff fixed point theorem, new multiplicity results for T-periodic solutions of unforced planar Hamiltonian systems $Ju' = \nabla_u H(t, u)$ and unforced undamped scalar second order equations $x'' + g(t, x) = 0$. In particular, by means of the Landesman–Lazer condition, we obtain sharp conclusions when the system is resonant at infinity.

1. Introduction

The problem of the existence of T-periodic solutions for the scalar second order equation

$$x'' + g(t, x) = 0, \tag{1.1}$$

with $g : [0, T] \times \mathbb{R} \to \mathbb{R}$, is a central topic in the theory of nonlinear ordinary differential equations. Focusing on the case when $g(t, x)$ has an at most linear growth in its second variable, it is well known that such a problem is strictly related to the interaction of $\frac{g(t,x)}{x}$ (for $|x|$ large) with the spectrum of the linear problem, defined as the set $\Sigma := \{\lambda_j\}_{j\in\mathbb{N}}$, where

$$\lambda_j := \left(\frac{2\pi j}{T}\right)^2.$$

In this context, several situations can occur. We recall the following three sufficient conditions of existence, which will be employed later on:

(a) the *nonresonance condition* given in [1], which generalizes the classical nonresonance assumption

$$\lambda_j < \liminf_{|x|\to+\infty} \frac{g(t, x)}{x} \le \limsup_{|x|\to+\infty} \frac{g(t, x)}{x} < \lambda_{j+1},$$

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