Nonlinear Analysis 74 (2011) 114–131

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Parabolic equations with nonlinear singularities

Pedro J. Martínez-Aparicio [a], Francesco Petitta [b,*]

[a] Departamento de Análisis Matemático, Campus Fuentenueva S/N, Universidad de Granada 18071, Granada, Spain

[b] Departamento de Análisis Matemático, Universitat de Valencia, C/ Dr. Moliner 50, 46100, Burjassot, Valencia, Spain

ARTICLE INFO

Article history:
Received 29 May 2010
Accepted 13 August 2010

MSC:
35K55
35K67
35B40

Keywords:
Nonlinear parabolic equations
Singular parabolic equations
Asymptotic behavior

ABSTRACT

We show the existence of positive solutions $u \in L^2(0,T;H_0^1(\Omega))$ for nonlinear parabolic problems with singular lower order terms of the asymptote-type. More precisely, we shall consider both semilinear problems whose model is

$$\begin{cases} u_t - \Delta u + \dfrac{u}{1-u} = f(x,t) & \text{in } \Omega \times (0,T), \\ u(x,0) = u_0(x) & \text{in } \Omega, \\ u(x,t) = 0 & \text{on } \partial\Omega \times (0,T), \end{cases}$$

and quasilinear problems having natural growth with respect to the gradient, whose model is

$$\begin{cases} u_t - \Delta u + \dfrac{|\nabla u|^2}{u^\gamma} = f(x,t) & \text{in } \Omega \times (0,T), \\ u(x,0) = u_0(x) & \text{in } \Omega, \\ u(x,t) = 0 & \text{on } \partial\Omega \times (0,T), \end{cases}$$

with $\gamma > 0$. Moreover, we prove a comparison principle and, as an application, we study the asymptotic behavior of the solution as t goes to infinity.

1. Introduction

In this paper we are concerned with both semilinear problems with an asymptote in the lower order term without any dependence on the gradient and quasilinear boundary value problems with lower order terms having quadratic dependence on the gradient and a singularity at $u = 0$.

Even if it were possible to consider more general singularities we will mainly focus our attention, for the sake of simplicity, on two problems, which turn out to be, in some sense, the extreme cases of a larger variety of problems. Due to the different nature of these problems we shall use completely different techniques to handle them, trying to give some insights on how the general case could be faced.

Specifically, we adapt some ideas of the elliptic results in [1,2] (see also [3–5]) to prove existence results in the parabolic case. However, as we shall see, the evolutionary setting forces us to handle many extra technical issues, most of them being completely by-passed in the stationary context.

In [2], the author considers an elliptic problem with an asymptote different from zero in the lower order term whose model is

* Corresponding author. Tel.: +34 963543937.
E-mail addresses: pedrojma@ugr.es (P.J. Martínez-Aparicio), francesco.petitta@uv.es (F. Petitta).

0362-546X/$ – see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2010.08.023