Nonlinear Analysis 74 (2011) 5815–5832

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Strongly amenable semigroups and nonlinear fixed point properties

Nicolas Bouffard

Department of Mathematical and Statistical Sciences, 632 CAB, University of Alberta, Edmonton, AB, T6G 2G1, Canada

ARTICLE INFO

Article history:
Received 24 February 2011
Accepted 28 March 2011
Communicated by Ravi Agarwal

Keywords:
Amenable semigroups
Semigroup compactifications
Minimal ideals
Left ideal groups
Non-expansive maps
Fixed point properties
Uniformly convex spaces
Left uniformly continuous functions
Ultimately non-expansive maps

ABSTRACT

A semi-topological semigroup is strongly left amenable if there is a compact left ideal group in the spectrum of its LUC-compactification. In this paper, we want to study those objects, and study some fixed point property related to non-expansive mapping and other similar kind of mapping.

1. Introduction

Amenability traces its origins back to the work of Lebesgue on finitely additive measures in 1904 [1], and to the work of Banach and Tarski on their well-known paradox [2,3]. The first study of this class of group was done by von Neumann in 1929 [4] as the class of non-paradoxical group. This class was later named *amenable* by Day which started the modern period of amenability in the 1940s, when amenability shifted from the study of invariant measures to the study of invariant means [5]. For a more detailed account of the history of amenability, see [6].

Extremely amenable semigroups were introduced by Mitchell [7,8] and Granirer [9] in the 60s. This is a very strong nonlinear property which is never possible for locally compact groups [10,11] except in the case of the trivial group. More recent development in the theory of extremely left amenable groups is presented in [12,13]. A weaker version of extreme amenability, namely n-extreme amenability, was introduced in 1970 by Lau [14,15]. The following year, Lau and Granirer proved that all locally compact n-extremely left amenable (n-ELA) group are finite groups, which was a major drawback since no non-trivial examples of n-ELA group were known. The first example of extremely left amenable topological group was given by Herer and Christensen in 1975 [16]. It is now known that extremely left amenable topological groups are quite common and many examples are known [12]. The concept of strong left amenability, which is our main interest in this paper, sits somewhere between n-extreme left amenability and left amenability.

In the first part of the paper (Sections 3–6) we study the structure of SLA semigroups, more specifically in Section 3, we give the definition of strong left amenability (SLA) and present some elementary results which act as the foundation to the following sections. Section 4 gives a characterization of SLA semigroups together with a justification for the use of the space $LUC(S)$ in our definition through the characterization of strong left amenability for other algebras. In Section 5, we show some ways to construct SLA semigroups and give a few examples. Finally, in Section 6, we characterize strong left amenability for some particular classes of semigroups which include discrete semigroups, and compact semigroups.

E-mail address: nb5@ualberta.ca.

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2011.03.052