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The viscous Cahn–Hilliard equation with inertial term

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ABSTRACT

We consider a hyperbolic relaxation of the viscous Cahn–Hilliard equation. This equation describes the early stages of spinodal decomposition in certain glasses. We establish the existence of families of exponential attractors and inertial manifolds which are continuous at any parameter of viscosity $\epsilon \geq 0$. Continuity properties of the global attractors are also examined.

1. Introduction

Many infinite-dimensional dynamical systems possess a finite-dimensional global attractor. This object is a compact set of the phase space which attracts uniformly the trajectories starting from bounded sets when time goes to infinity and thus appears as a suitable object for study in relation to the asymptotic behavior of such systems (see, e.g., [1]). Exponential attractors are compact and positively invariant sets with finite fractal dimension which attract all the trajectories starting from bounded sets at a uniform exponential rate. Clearly, an exponential attractor contains the global attractor. In addition, exponential attractors are more robust than global attractors with respect to perturbations and approximations (see [2–4] and its references). The finite dimensionality of the global attractor leads one to consider the question of whether there is a natural way of reconstructing the dynamics on the attractor as a finite-dimensional dynamics without direct recourse to the underlying evolution equation. The global attractor may have a complicated fractal structure, even for finite-dimensional dynamical systems, and a reasonably explicit description of the dynamics of the attractor might be out of reach. An answer to this question is given by the theory of inertial manifolds (see [5,1,6,7]). An inertial manifold is a positively invariant smooth finite-dimensional manifold which contains the global attractor and which attracts the trajectories at a uniform exponential rate. These features entail that the *a priori* infinite-dimensional dynamical system reduces, on the inertial manifold, to a finite system of ordinary differential equations.

The viscous Cahn–Hilliard equation with inertial term

$$\sigma \rho_{tt} + \rho_t - \Delta \left(\epsilon \rho_t - \Delta \rho + f'(\rho)\right) = 0, \tag{1.1}$$

where $\sigma, \epsilon > 0$, describes the early stages of spinodal decomposition in certain glasses (see [8,9]).

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