



# Existence of solutions of non-autonomous second order functional differential equations with infinite delay

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## ABSTRACT

In this paper the existence of solutions of a non-autonomous abstract retarded functional differential equation of second order with infinite delay is considered. Assuming the existence of an evolution operator corresponding to the associate abstract Cauchy problem of second order, we establish the existence of mild solutions of the functional equation. Furthermore, we study the existence of classical solutions of the abstract Cauchy problem of second order and we apply these results to establish the existence of classical solutions of the functional equation. Finally, we apply our results to study the existence of solutions of the non-autonomous wave equation with delay.

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## 1. Introduction

This paper is devoted to the study of the existence of mild and classical solutions for initial value problems described as a second order non-autonomous abstract retarded functional differential equations with infinite delay.

The theory of retarded functional differential equations has been the object of many works in the past decades. For the background on the theory we refer the reader to books [1–4]. In particular, last years there has been an increasing interest to study the partial functional differential equations. Throughout this paper,  $X$  denotes a Banach space endowed with a norm  $\|\cdot\|$ . The book of Wu [4] contains the basic theory for retarded functional differential equations with finite delay that can be described in the form

$$x'(t) = Ax(t) + f(t, x_t), \quad t \geq \sigma,$$

where the values  $x(t)$  belong to the Banach space  $X$  and  $A$  is the infinitesimal generator of a strongly continuous semigroup on  $X$ . Equations of this type will be called abstract retarded functional differential equations (ARFDE). Similar equations with infinite delay have been considered in many papers. We refer the reader to some of the most recent [5–16] and the references cited therein. Besides, it is well known [17] that the behavior of the first and second order abstract Cauchy problems is different in many aspects. For this reason, in [14,18] we have studied the existence of solutions for the second order ARFDE with infinite delay

$$x''(t) = Ax(t) + f(t, x_t), \quad t \geq \sigma,$$

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