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Strong convergence of iterative methods by strictly pseudocontractive mappings in Banach spaces

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ABSTRACT

In this paper we deal with fixed point computational problems by strongly convergent methods involving strictly pseudocontractive mappings in smooth Banach spaces. First, we prove that the S-iteration process recently introduced by Sahu in [14] converges strongly to a unique fixed point of a mapping T, where T is κ-strongly pseudocontractive mapping from a nonempty, closed and convex subset C of a smooth Banach space into itself. It is also shown that the hybrid steepest descent method converges strongly to a unique solution of a variational inequality problem with respect to a finite family of λ_i-strictly pseudocontractive mappings from C into itself. Our results extend and improve some very recent theorems in fixed point theory and variational inequality problems. Particularly, the results presented here extend some theorems of Reich (1980) [1] and Yamada (2001) [15] to a general class of λ-strictly pseudocontractive mappings in uniformly smooth Banach spaces.

1. Introduction

It is well known that if C is a nonempty, closed and convex subset of a Hilbert space H, then the nearest point projection P_C from H onto C is the unique sunny nonexpansive retraction of H onto C. This is not true for all Banach spaces, since outside Hilbert space, nearest point projections, although sunny, are no longer nonexpansive. Thus, the following interesting problem arises.

Problem 1.1. For which subsets of a Banach space does a sunny nonexpansive retraction exist?

The first result in this direction in a uniformly smooth Banach space was established by Reich in [1]. The result of Reich can be restated for nonexpansive mappings with bounded domain as follows.

Theorem 1.2. *Let C be a nonempty, closed, convex and bounded subset of a uniformly smooth Banach space X and let $T : C \to C$ be a nonexpansive operator. Then $F(T) := \{x \in C : x = Tx\}$ is nonempty sunny nonexpansive retract of C. Moreover, if $u \in C$ and z_t is the unique point in C defined by*

$$z_t = tu + (1-t)Tz_t, \quad t \in (0,1), \tag{1.1}$$

then $\{z_t\}$ converges strongly as $t \to 0^+$ to $Q_{F(T)}(u)$, where $Q_{F(T)}$ is the sunny nonexpansive retraction from C onto $F(T)$.

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