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On the method of alternating resolvents

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To Professor Dorin Ieşan on the occasion of his 70th birthday

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ABSTRACT

The work of Hundal [H. Hundal, An alternating projection that does not converge in norm, Nonlinear Anal. 57 (1) (2004) 35–61] has revealed that the sequence generated by the method of alternating projections converges weakly, but not strongly in general. In this paper, we present several algorithms based on alternating resolvents of two maximal monotone operators, A and B, that can be used to approximate common zeros of A and B. In particular, we prove that the sequences generated by our algorithms converge strongly. A particular case of such algorithms enables one to approximate minimum values of certain convex functionals.

1. Introduction and preliminaries

Let H be a real Hilbert space with scalar product $\langle \cdot, \cdot \rangle$ and induced norm $\| \cdot \|$. An operator $A : D(A) \subset H \to 2^H$ is called monotone if it satisfies the monotonicity property

$$\langle x - x', y - y' \rangle \geq 0, \quad \forall (x, y), (x', y') \in \text{graph}(A).$$

Equivalently, A is monotone if its graph is a monotone subset of the product space $H \times H$. If there is no monotone operator A' whose graph properly contains the graph of A, then A is called a maximal monotone operator. For a maximal monotone operator A, the resolvent of A, defined by $J_\beta^A := (I + \beta A)^{-1}$, is well defined on the whole space H and is single valued for every $\beta > 0$. Most importantly, J_β^A is nonexpansive; that is, for every $x, y \in H$, the inequality $\|J_\beta^A x - J_\beta^A y\| \leq \|x - y\|$ holds. See, for example, [1] for details.

We will use the following notations: given a sequence $(x_n)_{n\in\mathbb{N}_0}$, $\mathbb{N}_0 = \{0, 1, \ldots\}$, (or (x_n) in short), and a point $x \in H$, $x_n \to x$ (respectively, $x_n \rightharpoonup x$) means that (x_n) converges strongly (respectively, weakly) to x. The weak ω-limit set of (x_n) will be denoted by $\omega_w((x_n))$. This set is defined as follows:

$$\omega_w((x_n)) = \left\{x \in H \mid x_{n_k} \rightharpoonup x \text{ for some subsequence } (x_{n_k})_{k\in\mathbb{N}_0} \text{ of } (x_n)_{n\in\mathbb{N}_0}\right\}.$$

The class of proper and convex functions from H into $(-\infty, \infty]$ will be denoted by $\Gamma(H)$. For any $\varphi \in \Gamma(H)$, the subdifferential (operator) $\partial\varphi : H \to H$ is defined by

$$\partial\varphi(x) = \{w \in H \mid \varphi(x) - \varphi(v) \leq \langle w, x - v \rangle \text{ for all } v \in H\}.$$

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