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Solvability conditions for non-local boundary value problems for two-dimensional half-linear differential systems

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ABSTRACT

In this paper, we consider two non-local boundary value problems for two-dimensional half-linear differential systems. We prove general Fredholm type theorems, which allow one to derive new efficient solvability criteria for the problems studied.

1. Statement of problem and formulation of main results

On the interval $[a, b]$, we consider the differential system

$$\begin{aligned} \frac{du_1}{dt} &= p_1(t)|u_2|^{\lambda_1} \operatorname{sgn} u_2 + q_1(t, u_1, u_2), \\ \frac{du_2}{dt} &= p_2(t)|u_1|^{\lambda_2} \operatorname{sgn} u_1 + q_2(t, u_1, u_2) \end{aligned} \tag{1.1}$$

subjected to one of the following boundary conditions,

$$\int_a^{a_0} u_1(s)\, d\alpha_1(s) = \gamma_1(u_1, u_2), \qquad \int_{b_0}^{b} u_1(s)\, d\alpha_2(s) = \gamma_2(u_1, u_2) \tag{1.2}$$

and

$$\int_a^{a_0} u_1(s)\, d\alpha_1(s) = \gamma_1(u_1, u_2), \qquad \int_{b_0}^{b} u_2(s)\, d\alpha_2(s) = \gamma_2(u_1, u_2). \tag{1.3}$$

In the case, where $\lambda_1 = \lambda_2 = 1$, problems (1.1), (1.2) and (1.1), (1.3) as well as their particular cases are studied in detail (see, e.g., [1–18] and the references therein). As for the case, where system (1.1) is half-linear, i.e., if

$$\lambda_1 > 0, \qquad \lambda_1 \neq 1, \qquad \lambda_1\lambda_2 = 1, \tag{1.4}$$

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