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Infinitely many solutions for diffusion equations without symmetry[☆]

Jun Wang[a,b,*], Junxiang Xu[b], Fubao Zhang[b]

[a] Department of Mathematics, Jiangsu University, Zhenjiang, Jiangsu, 212013, China
[b] Department of Mathematics, Southeast University, Nanjing, Jiangsu, 210096, China

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ABSTRACT

We consider the following diffusion system:

$$\begin{cases} \partial_t u - \triangle_x u + b(t,x)\nabla_x u + V(x)u = H_v(t,x,u,v), \\ -\partial_t v - \triangle_x v + b(t,x)\nabla_x v + V(x)v = H_u(t,x,u,v) \end{cases} \quad \forall\,(t,x)\in\mathbb{R}\times\mathbb{R}^N,$$

which is an unbounded Hamiltonian system in $L^2(\mathbb{R}\times\mathbb{R}^N,\mathbb{R}^{2m})$, $z:=(u,v):\mathbb{R}\times\mathbb{R}^N\to\mathbb{R}^m\times\mathbb{R}^m$, $b\in C(\mathbb{R}\times\mathbb{R}^N,\mathbb{R}^N)$, $V\in C(\mathbb{R}^N,\mathbb{R})$ and $H\in C^1(\mathbb{R}\times\mathbb{R}^N\times\mathbb{R}^{2m},\mathbb{R})$. Suppose that H, b and V depend periodically on t and x, and that $H(t,x,z)$ is superquadratic in z as $|z|\to\infty$. Without a symmetry assumption on H, we establish the existence of infinitely many geometrically distinct solutions via a variational approach.

1. Introduction and main results

1.1. The problem

In this paper, we study the multiplicity of solutions for the following system:

$$(HS)\quad \begin{cases} \partial_t u - \triangle_x u + b(t,x)\nabla_x u + V(x)u = H_v(t,x,u,v), \\ -\partial_t v - \triangle_x v + b(t,x)\nabla_x v + V(x)v = H_u(t,x,u,v) \end{cases} \quad \forall\,(t,x)\in\mathbb{R}\times\mathbb{R}^N.$$

Here $V\in C(\mathbb{R}^N,\mathbb{R})$, $b:=(b_1,\ldots,b_N)\in C(\mathbb{R}\times\mathbb{R}^N,\mathbb{R}^N)$ with the gauge condition $\mathrm{div}b(t,x):=\sum_{i=1}^N \partial_{x_i}b_i(t,x)=0$, $z=(u,v):\mathbb{R}\times\mathbb{R}^N\to\mathbb{R}^m\times\mathbb{R}^m$ and $H\in C^1(\mathbb{R}\times\mathbb{R}^N\times\mathbb{R}^{2m},\mathbb{R})$. Such problems arise in the optimal control of systems governed by partial differential equations (see [1]) and they are related to Schrödinger equations (see [2]). If we let

$$\mathcal{J}_0=\begin{pmatrix}0 & I\\ I & 0\end{pmatrix},\qquad \mathcal{J}=\begin{pmatrix}0 & -I\\ I & 0\end{pmatrix},\qquad \mathcal{S}:=-\triangle_x+V, \tag{1.1}$$

and

$$A=\mathcal{J}_0\mathcal{S}+\mathcal{J}b\cdot\nabla_x, \tag{1.2}$$

then (HS) can be rewritten as

$$\mathcal{J}\frac{\mathrm{d}}{\mathrm{d}t}z+Az=H_z(t,x,z). \tag{1.3}$$

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* Corresponding author at: Department of Mathematics, Southeast University, Nanjing, Jiangsu, 210096, China.
E-mail addresses: wangjdn2006@21cn.com (J. Wang), xujun@seu.edu.cn (J.X. Xu), zhangfubao@seu.edu.cn (F.B. Zhang).

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