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On boundary blow-up solutions to equations involving the ∞-Laplacian

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ABSTRACT

Given a non-negative, continuous function h on $\overline{\Omega} \times \mathbb{R}$ such that $h(x, 0) = 0$ for all $x \in \Omega$, $h(x, t) > 0$ in $\Omega \times (0, \infty)$, and $h(x, t)$ non-decreasing in t for each $x \in \Omega$, we study the boundary value problem

$$\begin{cases} \Delta_\infty u = h(x, u) & \text{in } \Omega \\ u = \infty & \text{on } \partial\Omega \end{cases}$$

where $\Omega \subseteq \mathbb{R}^N$, $N \geq 2$ is a bounded domain and Δ_∞ is the ∞-Laplacian, a degenerate elliptic operator. We provide sufficient conditions on h under which the above problem admits a solution, or fails to admit a solution. A necessary and sufficient condition on f is given for a solution to exist in the special case when $h(x, t) = b(x)f(t)$. In the latter case an asymptotic boundary behavior of solutions will be studied. As an application a sufficient condition on f will be given to ensure the uniqueness of solutions in case b is a constant.

1. Introduction

Let $\Omega \subseteq \mathbb{R}^N$, $N \geq 2$ be a bounded domain. In this paper we study the problem

$$\begin{cases} \Delta_\infty u = h(x, u) & \text{in } \Omega \\ u = \infty & \text{on } \partial\Omega. \end{cases} \tag{1.1}$$

The boundary condition $u = \infty$ on $\partial\Omega$ is to be understood in the following sense.

$$\lim_{x \to z} u(x) = \infty \quad \text{for each } z \in \partial\Omega.$$

Here Δ_∞ is the degenerate elliptic operator given by

$$\Delta_\infty u = \sum_{i,j=1}^{N} u_{x_i} u_{x_i x_j} u_{x_j}.$$

This operator, called the ∞-Laplacian, first appeared in the work of Aronsson [1] in connection with the geometric problem of finding the so-called absolutely minimizing functions in Ω. We refer the reader to the papers [2–4] for an in-depth discussion of the problem and other many interesting related properties.

The ∞-Laplacian has been the subject of extensive investigation since the fundamental work of Jensen [5] in which he established that solutions of $\Delta_\infty u = 0$ in Ω characterize absolutely minimizing functions in Ω. Since absolutely minimizing functions are not necessarily smooth, the equation $\Delta_\infty u = 0$ in Ω is understood in a weak sense to be described in Section 2.

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