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The reversibility and the center problem☆

Jaume Giné*, Susanna Maza

Departament de Matemàtica, Universitat de Lleida, Avda. Jaume II, 69, 25001, Lleida, Spain

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ABSTRACT

In this work we study the narrow relation between reversibility and the center problem and also between reversibility and the integrability problem. It is well known that an analytic system having either a non-degenerate or nilpotent center at the origin is analytically reversible or orbitally analytically reversible, respectively. In this paper we prove the existence of a smooth map that transforms an analytic system having a degenerate center at the origin with either an analytic first integral or a $\mathcal{C}^\infty$ inverse integrating factor into a reversible linear system (after rescaling the time). Moreover, if the degenerate center has an analytic or a $\mathcal{C}^\infty$ reversing symmetry, then the transformed system by the map also has a reversing symmetry. From the knowledge of a first integral near the center we give a procedure to detect reversing symmetries.

1. Introduction

This paper is focused on the planar differential systems with a reversing symmetry. A reversing symmetry is one of the fundamental symmetries in natural science and it arises in many branches in physics; see for instance [1–3] and the references therein. More specifically, this work is devoted to the study of two problems arising in the theory of analytic reversible systems in the plane. The first one is the study of the narrow relation between reversibility and the center problem; see [4,5]. It is well known that an analytic system having a non-degenerate (resp. nilpotent) center at the origin is analytically reversible (resp. orbitally analytically reversible); see [6–8]. Nevertheless the relation between reversibility and the center problem for degenerate singular points is not established. In this paper we prove the existence of a smooth map that transforms an analytic system having a degenerate center at the origin with either an analytic first integral or a $\mathcal{C}^\infty$ inverse integrating factor into a reversible linear system (after rescaling the time). Moreover, if the degenerate center has an analytic or a $\mathcal{C}^\infty$ reversing symmetry, the transformed system by the map also has a reversing symmetry. From the knowledge of a local smooth first integral near the center we give a procedure to detect reversing symmetries in some cases. The second problem on which we focus our attention is the study of the existence of a local analytic first integral for a reversible system in a neighborhood of a non-degenerate singular point and also for certain degenerate singular points.

The paper is organized as follows: In Section 2 we summarize some general definitions and results about reversible systems. In Section 3 we study the relation between reversibility and the center problem presenting our results about the reversibility and the center problem for degenerate singular points. In Section 4 we show that an orbitally analytically reversible system always has a local analytic first integral in a neighborhood of a non-degenerate singular point. Moreover, we prove the existence of a local analytic first integral in a neighborhood of a degenerate singular point for some reversible systems. In the last section we give the proofs of all the results presented in this work.

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* Corresponding author. Tel.: +34 973702778; fax: +34 973702702.
E-mail addresses: gine@matematica.udl.cat (J. Giné), smaza@matematica.udl.cat (S. Maza).

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