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Global solution branches for equations involving nonhomogeneous operators of p-Laplace type

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ABSTRACT

It is shown that if μ is not an eigenvalue of an associated p-Laplacian, then the equation

$$-\operatorname{div}(\varphi(x, \nabla u)) = \mu\, |u|^{p-2}\, u + f(\lambda, x, u, \nabla u)$$

with nonhomogeneous φ (which is assumed to behave asymptotically as the function generating the associated p-Laplacian) has a global branch of solutions (λ, u). Also the case of modified p-Laplace operators and generalizations thereof are discussed.

1. Introduction

Using abstract nonlinear spectral theory and the theory of essential (0-epi) maps in the spirit of [1–3], the existence of a global branch of solutions (λ, u) for the boundary value problem

$$\begin{cases} -\Delta_p u = \mu\, |u|^{p-2}\, u + f(\lambda, x, u, \nabla u) & \text{on } \Omega, \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1.1}$$

was obtained in [4] when $1 < p < \infty$ (for generalizations to unbounded domains and with weight functions, see also [5,6]). Here, we denote by Δ_p either the p-Laplacian

$$\Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\, \nabla u) \tag{1.2}$$

or the modified p-Laplacian

$$\Delta_p u := \sum_{k=1}^{n} \frac{\partial}{\partial x_k} \left(\left| \frac{\partial u}{\partial x_k} \right|^{p-2} \frac{\partial u}{\partial x_k} \right). \tag{1.3}$$

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