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Traveling waves for a nonlocal anisotropic dispersal equation with monostable nonlinearity

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ABSTRACT

This paper is concerned with traveling wave solutions of the equation

$$\frac{\partial u}{\partial t} = J * u - u + f(u) \quad \text{on } \mathbb{R} \times (0, \infty),$$

where the dispersion kernel J is nonnegative and the nonlinearity f is monostable type. We show that there exists $c^* \in \mathbb{R}$ such that for any $c > c^*$, there is a nonincreasing traveling wave solution ϕ with $\phi(-\infty) = 1$ and $\lim_{\xi\to\infty} \phi(\xi)e^{\lambda\xi} = 1$, where $\lambda = \Lambda_1(c)$ is the smallest positive solution to $c\lambda = \int_{\mathbb{R}} J(z)e^{\lambda z}dz - 1 + f'(0)$. Furthermore, the existence of traveling wave solutions with $c = c^*$ is also established. For $c \neq 0$, we further prove that the traveling wave solution is unique up to translation and is globally asymptotically stable in certain sense.

1. Introduction and main results

In this paper, we shall consider traveling wave solutions of a nonlocal dispersal equation of the form

$$\frac{\partial u}{\partial t} = J * u - u + f(u), \tag{1.1}$$

where the kernel J of the convolution $(J * u)(x, t) = \int_{\mathbb{R}} J(x - y)u(y, t)dy$ is a nonnegative function of mass one and the nonlinearity f is monostable type.

When the kernel J is symmetric, Eq. (1.1) can be seen as a nonlocal analogue of the following reaction–diffusion equation

$$\frac{\partial u}{\partial t} = \Delta u + f(u) \tag{1.2}$$

in one-dimensional spatial space. If $f(u) = u(1 - u)$, then Eq. (1.2) reduces to the well-known Fisher equation

$$\frac{\partial u}{\partial t} = \Delta u + u(1 - u),$$

which was introduced by Fisher [1] in 1930 in order to model the spatial spread of an advantageous form of a single gene in a population, where u represents the gene fraction of the mutant, the dispersion of the genetic characters is assumed to

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